HW 7 Solutions

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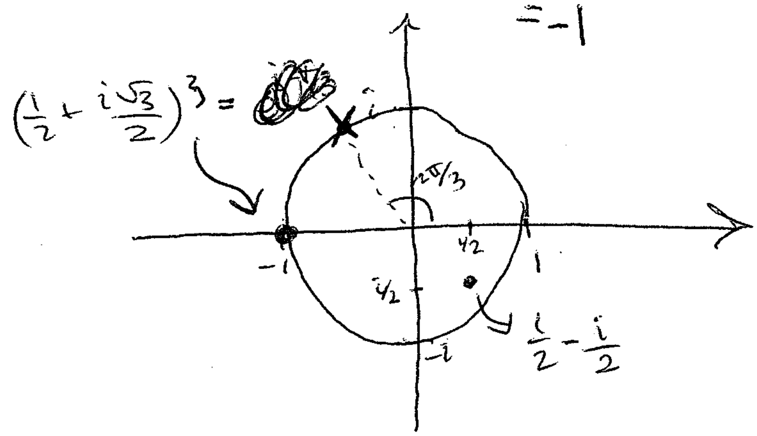
- $\left(\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)^3 \overset{\text{polar}}{=} \left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^3$

$$\overset{\text{de Moivre}}{=} \cos\left(3\cdot\frac{\pi}{3}\right)+i\sin\left(3\cdot\frac{\pi}{3}\right)$$

$$= -1$$



so modulus $\underline{\underline{1}}$

arg $\underline{\underline{\pi}}$

- $\frac{1}{1+i}\cdot\frac{1-i}{1-i} = \frac{1-i}{1-i^2} = \frac{1-i}{2} = \frac{1}{2}-\frac{i}{2}$

modulus $= \sqrt{\left(\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)^2} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = \underline{\underline{\frac{\sqrt{2}}{2}}}$

arg $= \arctan\left(\frac{-1/2}{1/2}\right) = \arctan(-1) = \underline{\underline{-\pi/4}}$ (or $7\pi/4$)

(note: IV quadrant)

264) (c) Is $\text{Im}(z)+\text{Im}(w)=\text{Im}(z+w)$?

Let $z=a+bi$, $w=c+di$.

Then $\text{Im}(z)+\text{Im}(w)=\underline{b+d}$, and

$\text{Im}(z+w)=\text{Im}\left((a+c)+(b+d)i\right)=\underline{b+d}$. $\boxed{\text{YES}}$

(d) Is $\overline{zw}=(\overline{z})(\overline{w})$?

z, w as above. Then $\overline{zw}=\overline{(a+bi)(c+di)}=$

$\overline{(ac-bd)+(bc+ad)i}=$

$(ac-bd)-(bc+ad)i$.

Also $(\overline{z})(\overline{w})=(a-bi)(c-di)=(ac-bd)-(bc+ad)i$. $\boxed{\text{YES}}$

266) Try to find $z=a+bi$. We're given that

$\text{Re}(z)=\frac{1}{2}|z|$ and $\text{Im}(z)=1$.

$\Downarrow$ $\Downarrow$

$a=\frac{1}{2}\left(\sqrt{a^2+b^2}\right)$ $b=1$

$a=\frac{1}{2}\sqrt{a^2+1}$

Square both sides: $a^2=\frac{1}{4}(a^2+1)$

so $\frac{3}{4}a^2=\frac{1}{4}$

$a^2=\frac{1}{3}$

$a=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$ (want + since modulus/absolute val. is +)

So $\boxed{z=\frac{\sqrt{3}}{3}+i}$

Some of

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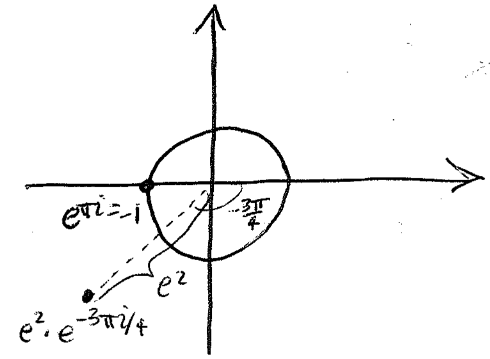
- $\dfrac{e^{(2-\pi i/2)}}{e^{\pi i/4}} = \dfrac{e^{2}\cdot e^{-\pi i/2}}{e^{\pi i/4}} = e^{2}\cdot e^{-\pi i/2}\cdot e^{-\pi i/4} = e^{2}\cdot e^{-3\pi i/4}$ (add exp)

  e^2 ↓ real number – no i in exp.

- $e^{2009\pi i} = e^{2008\pi i}\cdot e^{\pi i} = e^{\pi i} = -1$ since even multiples of π are the same as 0



279 | (f) $3z^6 = z^3 + 2$, or $3z^6 - z^3 - 2 = 0$

$= 3(z^3)^2 - z^3 - 2 = 0$

quadratic in the variable z^3.

So use quadr. form.: $z^3 = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-2)}}{6}$

$= \dfrac{1 \pm 5}{6} = \{1, -\frac{2}{3}\}$

for $z^3 = 1 = e^{i(0+2\pi k)}$, take cube root:

$z = (e^{i2\pi k})^{1/3} = e^{i\frac{2\pi k}{3}}$

$k=0 \quad k=1 \quad k=2$

So $z = \{1, e^{i\frac{2\pi}{3}}, e^{i\frac{4\pi}{3}}\}$

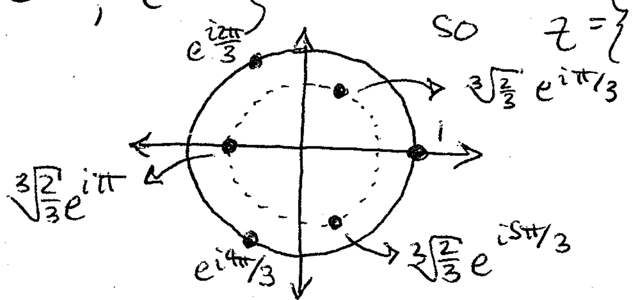
for $z^3 = -\frac{2}{3} = \frac{2}{3}e^{i(\pi+2k\pi)}$,

$z = \left[\frac{2}{3}e^{i(\pi+2k\pi)}\right]^{1/3}$

$= \underbrace{\sqrt[3]{\tfrac{2}{3}}}_{\text{(real)}}\, e^{i\left(\frac{\pi+2k\pi}{3}\right)}$

$k=0 \quad k=1 \quad k=2$

so $z = \{\sqrt[3]{\tfrac{2}{3}}\, e^{i\pi/3}, \sqrt[3]{\tfrac{2}{3}}\, e^{i\pi}, \sqrt[3]{\tfrac{2}{3}}\, e^{i\frac{5\pi}{3}}\}$



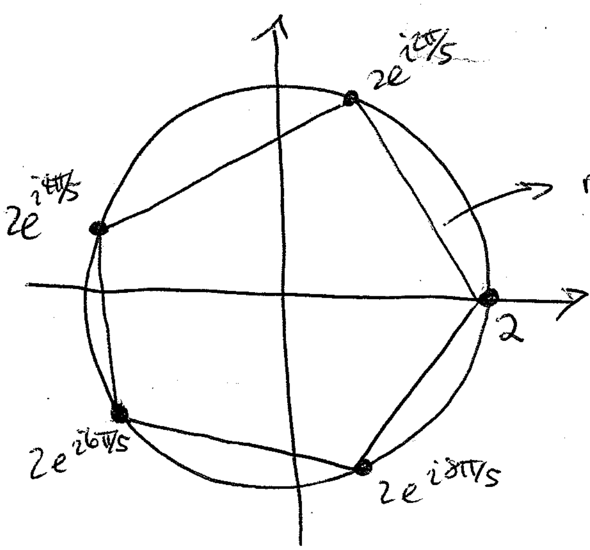
Note $\sqrt[3]{\tfrac{2}{3}} < 1$

2791 (g) $z^5 - 32 = 0$, or $z^5 = 32 = 32e^{i(0+2k\pi)}$

so $z = \underbrace{\sqrt[5]{32}}_{2^{\prime\prime}} \cdot e^{\frac{i2k\pi}{5}}$

so $z = \{2, 2e^{\frac{i2\pi}{5}}, 2e^{\frac{i4\pi}{5}}, 2e^{\frac{i6\pi}{5}}, 2e^{\frac{i8\pi}{5}}\}$

$k=0$, 1, 2, 3, 4 (note $k=5$ gives $2e^{\frac{i10\pi}{5}} = 2$)



regular pentagon