Selected Solutions for HW 1 :

2. $\int x/a + a/x + x^a + a^x + ax \quad dx$

$$= \boxed{\frac{1}{a}\frac{x^2}{2} + a\ln|x| + \frac{x^{a+1}}{a+1} + \frac{a^x}{\ln(a)} + a\frac{x^2}{2} + C}$$

11. $\int_1^2 \frac{x^2+1}{\sqrt{x}}\,dx = \int_1^2 (x^2+1)x^{-1/2}\,dx$

$$= \int_1^2 x^{3/2} + x^{-1/2}\,dx$$

$$= \left(\frac{2}{5}x^{5/2} + 2x^{1/2}\right)\Big|_1^2$$

$$= \frac{2}{5}(2)^{5/2} + 2(2)^{1/2} - \left(\frac{2}{5} + 2\right)$$

$$= \boxed{\frac{18}{5}\cdot\sqrt{2} - \frac{12}{5}} \quad (\approx 2.69\ldots)$$

25. $\int_0^{\pi/2} \cos\theta + \sin 2\theta \; d\theta$

$$= \sin\theta + \frac{-1}{2}\cos(2\theta)\Big|_0^{\pi/2}$$

because $\frac{d}{d\theta}\sin\theta = \cos\theta$

and $\frac{d}{d\theta}\cos 2\theta = -2\sin 2\theta$

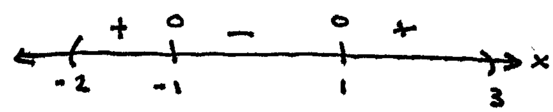
$$= \left(\sin\left(\frac{\pi}{2}\right) - \frac{\cos(\pi)}{2}\right) - \left(\sin(0) - \frac{\cos(0)}{2}\right)$$

$$= 1 - \left(\frac{-1}{2}\right) - \left(0 - \frac{1}{2}\right) = \boxed{2}$$

34. Calculate $\int_{-2}^{3} |x^2-1|\,dx$.

ans. First I'm going to write $|x^2-1|$ as a piecewise function:

$(x^2-1) = (x+1)(x-1)$
- has roots ± 1
- is positive when $x < -1$ and $x > 1$
- is negative when $-1 < x < 1$



So $$|x^2-1| = \begin{cases} x^2-1 & x \le -1 \\ -(x^2-1) & -1 < x < 1 \\ x^2-1 & x \geq 1 \end{cases}$$

Now we can write

$$\int_{-2}^{3} |x^2-1|\,dx = \int_{-2}^{-1} |x^2-1|\,dx + \int_{-1}^{1} |x^2-1|\,dx + \int_{1}^{3} |x^2-1|\,dx$$

$$= \int_{-2}^{-1} x^2-1\,dx + \int_{-1}^{1} -(x^2-1)\,dx + \int_{1}^{3} x^2-1\,dx$$

$$= \left(\frac{x^3}{3}-x\right)\Big|_{-2}^{-1} - \left(\frac{x^3}{3}-x\right)\Big|_{-1}^{1} + \left(\frac{x^3}{3}-x\right)\Big|_{1}^{3}$$

$$= \left[\left(-\frac{1}{3}+1\right) - \left(-\frac{8}{3}+2\right)\right] - \left[\left(\frac{1}{3}-1\right) - \left(-\frac{1}{3}+1\right)\right] + \left[\left(\frac{3^3}{3}-3\right) - \left(\frac{1}{3}-1\right)\right]$$

$$= \left[\frac{2}{3} - \left(-\frac{2}{3}\right)\right] - \left[-\frac{2}{3} - \frac{2}{3}\right] + \left[9-3 - \left(-\frac{2}{3}\right)\right]$$

$$= \frac{4}{3} + \frac{4}{3} + 6 + \frac{2}{3} = \boxed{28/3} \quad (= 9\tfrac{1}{3})$$

40. Compute $I = \int_0^2 2x(1+x^2)^3\,dx$ in two ways

(i) Expand :

ans First, $(1+x^2)^3 = 1 + 3x^2 + 3x^4 + x^6$

so $2x(1+x^2)^3 = 2(x + 3x^3 + 3x^5 + x^7)$

Thus

$$I = \int_0^2 2x(1+x^2)^3\,dx$$

$$= 2\int_0^2 x + 3x^3 + 3x^5 + x^7\ dx$$

$$= 2\left(\frac{x^2}{2} + 3\frac{x^4}{4} + 3\frac{x^6}{6} + \frac{x^8}{8}\right)\Big|_0^2$$

$$= 2\left(\frac{4}{2} + 3\frac{2^4}{4} + 3\frac{2^6}{6} + \frac{2^8}{8} - 0\right)$$

$$= 2(2 + 3\cdot 4 + 2^5 + 2^5) = 2(2+12+64)$$

$$= 2\cdot 78 = \boxed{156}$$

(ii) sub $u = 1 + x^2$:

ans let $u = 1+x^2$, so $du = 2x\,dx$

Thus $\int 2x(1+x^2)^3\,dx = \int u^3\,du = \frac{u^4}{4} + c = \frac{(1+x^2)^4}{4} + c$

So $I = \int_0^2 2x(1+x^2)^3\,dx = \frac{(1+x^2)^4}{4}\Big|_0^2 = \frac{5^4}{4} - \frac{1}{4}$

$$= \frac{625-1}{4}$$

$$= \boxed{156}$$

(Yay! Should get the same answer in (i) and (ii).)