Mathematics 234

Review problems:

1. Assume the differentiable function of three variables, $f(x,y,z)$, satisfies

$$f(1,-2,2)=6, \qquad f_x(1,-2,2)=2,\ f_y(1,-2,2)=1,\ f_z(1,-2,2)=-3.$$

Using this information, answer the following questions:

(a) Compute the value of the directional derivative of f at $P=(1,-2,2)$ in the direction toward the origin.

(b) Suppose you are told that there is a direction u in which the directional derivative of f equals 4. Can this be true? If so, find such a direction u. If not, explain why not.

(d) Suppose the point $(1,-2,2)$ is contained in the level set $E:=\{(x,y,z): f(x,y,z)=6\}$. Find the tangent plane to E at $(1,-2,2)$ and a parametrization th the line normal to E.

(e) Let S be the sphere of radius 3 centered at the origin. Find a line which is both tangential to E and tangential to S at $(1,-2,2)$.

(f) A particle moving along a curve C parametrized by

$$\mathbf{r}(t)=(t-1)\mathbf{i}+(t^2-6)\mathbf{j}+(t^3-6)\mathbf{k}$$

sees changing values of $f(x,y,z)$. Compute the rate of change $\frac{d}{dt}\big(f(\mathbf{r}(t)\big)$, at time $t=2$.

2. Consider the surface defined by

$$(*) \qquad x^2+y^2-z^2=2x(y+z)-2.$$

(a) Find the equation of the plane tangent to the surface at the point $P=(1,2,1)$ on S.

(b) The equation (*) for S defines z as a function of x and y, i.e. $z=g(x,y)$. Compute the partial derivatives $\frac{\partial g}{\partial y}(1,2)$, $\frac{\partial g}{\partial x}(1,2)$ *using implicit differentiation.*

(c) What is the linearization of g at $(1,2)$?

(d) Compute the second derivatives g_{xx}, g_{xy}, g_{yx} and g_{yy} *using implicit differentiation.*

(e) Let $\vec{u}$ be a direction and denote by D_u the directional derivative with respect to $\vec{u}$. Compute the second directional derivative $D_u^2 g(1,2):=D_uD_ug(1,2)$.