

Mathematics 629

Math 629. Homework due Wednesday April 28, 2010.

I. Problem 64 page 80 in Folland's book (do this especially if you have not done it before)..

II. For $n = 2, 3, \dots$, and $0 < R < 1$, let $A_n(R)$ be the $(n - 1)$ dimensional area of the region in the unit sphere in \mathbb{R}^n defined by $x_1^2 + x_2^2 + \dots + x_{n-1}^2 < R^2$. Let $V_n(R)$ be the volume of the region in the unit ball defined by the same inequality.

1) Evaluate $A_2(R)$, $V_2(R)$, $A_3(R)$ and $V_3(R)$.

2) Give the values of $A_n(R)$ and $V_n(R)$ in terms of simple integrals that you are not asked to evaluate.

III. Let $E \subset \mathbb{R}^2$ be a measurable set with the following property: Every line through the origin contains a line segment of length 1 which lies in E .

Let A be the infimum over the Lebesgue measures of sets E with this property. Can you compute A ? Is there a measurable set E such that $m(E) = A$?