Mathematics 629
A little quiz

1. Assuming you know what a Lebesgue measurable set is, fill in the following:
(i) A function $f : \mathbb{R} \to \mathbb{R}$ is Lebesgue measurable if ...

(ii) A function $f : \mathbb{R} \to \mathbb{R}$ is integrable if ...

(iii) A sequence of Lebesgue measurable functions $f_n : \mathbb{R} \to \mathbb{R}$ converges in measure if ...

2. Let f_n be a sequence of real-valued Lebesgue measurable functions defined on $[0, 1]$. Answer the following questions and justify your answer either with a counterexample or proof.
(i) If f_n converges in measure, does f_n necessarily converge almost everywhere?

(ii) If $f_n : [0, 1] \to \mathbb{R}$ converges almost everywhere, does f_n necessarily converge in measure?

(iii) Now change $[0, 1]$ to the full real line: If $f_n : \mathbb{R} \to \mathbb{R}$ is Lebesgue measurable on $\mathbb{R}$ and if f_n converges almost everywhere, does f_n necessarily converge in measure on $\mathbb{R}$?

3. Let $f : \mathbb{R} \to \mathbb{R}$ be Lebesgue measurable with $\int_{\mathbb{R}} |f| dm < \infty$. Prove that for every $\epsilon > 0$ there is a $\delta > 0$ so that $\left| \int_E f dm \right| < \epsilon$ holds for all measurable subsets of $\mathbb{R}$ with $m(E) < \delta$.