Due Monday, April 3

When nothing else is specified, the exercise takes place on some abstract measure space \((X, \mathcal{M}, \mu)\).

1. Given a function \(f : \mathbb{R} \rightarrow \mathbb{R}\) and \(h \in \mathbb{R}\), define the translated function \(f_h\) by \(f_h(x) = f(x + h)\). Suppose \(f\) is Borel measurable and integrable under Lebesgue measure. Show that then \(f_h\) is also Borel measurable and integrable. Show that this translation operation is continuous in the following sense:

\[
\lim_{h \to 0} \int_{\mathbb{R}} |f_h - f| \, dx = 0.
\]

2. Exercise 34 on page 63. Notice that what you are proving here is a dominated convergence theorem under an assumption of convergence in measure. (It is not necessary to prove part a. first.)