831 Theory of Probability Fall 2009 Homework 6

Due Thursday, November 12

1. IID variables close up. Let $\{Y_k\}$ be i.i.d. random variables on $\mathbb{R}$ with a common *continuous* density f. As we put more and more points Y_k down they tend to concentrate so let us spread them out by multiplying by n, and also look at them around the point nc, for some fixed $c \in \mathbb{R}$. In other words, for each $n \in \mathbb{N}$ define

$$X_{n,k} = n(Y_k - c), \quad 1 \leq k \leq n.$$

Let $N_n(a,b) = \sum_{k=1}^{n} \mathbf{1}_{(a,b)}(X_{n,k})$ be the number of $X_{n,k}$ that fall into (a,b). Find a weak limit for $N_n(a,b)$ as $n \to \infty$.

2. Exercises 1.3, 1.6 and 1.7 on p. 174–175.

3. Let $0 < p < 1$ and let S_n be the simple random walk with step probabilities $p, 1-p$, in other words $S_n = X_1 + \cdots + X_n$ and the $\{X_i\}$ are i.i.d. random variables with distribution

$$P(X_i = 1) = p \qquad \text{and} \qquad P(X_i = -1) = 1 - p.$$

Fix a positive integer $b > 0$, and let T be the first hitting time of the point b:

$$T(\omega) = \inf\{n \geq 1 : S_n(\omega) = b\}.$$

If the walk never hits b, in other words the set $\{n : S_n(\omega) = b\}$ is empty, then $T(\omega) = \infty$. Calculate the expectation ET. Be sure to justify everything.