Answer to Timo's second question

Let K_{Airy} denote the Airy operator acting on all of $L^2(\mathbb{R})$ and P_t the operator of multiplication by $\chi_{(t,\infty)}$, so

$$F_2(t) = \det(I - P_t\, K_{\text{Airy}}\, P_t).$$

We'll show that K_{Airy} is a projection operator (it's self-adjoint and $K_{\text{Airy}}^2 = K_{\text{Airy}}$), so it's spectrum is $\{0, 1\}$, and it follows that the spectrum of $P_t\, K_{\text{Airy}}\, P_t$ is $\subset [0, 1]$. Moreover, if the eigenvalues of $P_t\, K_{\text{Airy}}\, P_t$ are

$$\lambda_1(t) \geq \lambda_2(t) \geq \cdots$$

then each $\lambda_i(t)$ increases as t decreases. This follows from the minimax characterization of the eigenvalues (also known as the Weyl-Courant lemma) because $\{P_t\}$ is an increasing family of projections as t decreases. Also,

$$\lambda_1(t) = \|P_t\, K_{\text{Airy}}\, P_t\| \to \|K_{\text{Airy}}\| = 1$$

as $t \to -\infty$ because $P_t \to I$ strongly as $t \to -\infty$. It follows that

$$\det(I - P_t\, K_{\text{Airy}}\, P_t) = \prod_{i=1}^{\infty} (1 - \lambda_i(t))$$

decreases as t decreases and

$$\det(I - P_t\, K_{\text{Airy}}\, P_t) \leq 1 - \lambda_1(t) \to 0 \text{ as } t \to -\infty.$$

So let's show that K_{Airy} is a projection. It's clear that it's self-adjoint, so we have to show that $K_{\text{Airy}}^2 = K_{\text{Airy}}$. We have

$$K_{\text{Airy}}(x, y) = \int_0^\infty \text{Ai}(x + z)\, \text{Ai}(z + y)\, dz,$$

so if P^+ denotes multiplication by $\chi_{\mathbb{R}^+}$ and A denotes the operator on $L^2(\mathbb{R})$ with kernel $\text{Ai}(x + y)$ then $K_{\text{Airy}} = A\, P^+\, A$.

Define J by $Jf(x) = f(-x)$. Since $J^2 = I$ and $J^* = J$, we have $A^2 = AJ \cdot JA = AJ(AJ)^*$. Now AJ is the operator with kernel $\text{Ai}(x-y)$ so in the Fourier transformed space (more precisely, conjugating with the Fourier transform) AJ becomes multiplication by the Fourier transform of Ai, which is $e^{i\xi^3/3}$. Therefore $AJ(AJ)^*$ becomes multiplication by $|e^{i\xi^3/3}|^2 = 1$. Therefore $A^2 = AJ(AJ)^* = I$, and so

$$K_{\text{Airy}}^2 = A\, P^+\, A^2\, P^+\, A = A\, {P^+}^2\, A = A\, P^+\, A = K_{\text{Airy}}.$$