Math 491 - Linear Algebra II, Fall 2016

Homework 8 - Jordan Form

Quiz on 4/12/16

Remark: Answers should be written in the following format:
A) Result.
B) If possible, the name of the method you used.
C) The computation or proof.

1. **Computing Jordan Forms.** For each of the following matrices, find a Jordan matrix J, to which it is similar. In each case, compute a Jordan basis for the appropriate vector space.

$$\begin{pmatrix} -4 & 6 & 0 \\ -3 & 5 & 0 \\ -3 & -6 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

2. **Classifying Matrices.** Find all matrices up to similarity in $M_{11}(\mathbb{Q})$ with minimal polynomial $m(x) = x(x-1)^2(x+4)^3$ and characteristic polynomial $p(x) = x^2(x-1)^5(x+4)^4$.

3. **Jordan Matrices and Similarity.**
   (a) How many Jordan matrices are there in $M_6(\mathbb{C})$ with minimal polynomial $m(x) = (x+2)^4(x-1)^2$.
   (b) How many matrices up to similarity are there in $M_6(\mathbb{C})$ with minimal polynomial $m(x) = (x+2)^4(x-1)^2$.

4. **A sufficient condition for similarity.** Let $A, B \in M_n(\mathbb{F})$ with $m_A(x) = m_B(x)$, and

$$p_A(x) = p_B(x) = (x-\lambda_1)^{d_1} \cdots (x-\lambda_k)^{d_k}.$$

   Suppose the $\lambda_i \in \mathbb{F}$ are distinct, and for all $1 \le i \le k$, $1 \le d_i \le 3$. Show that A and B are similar.

5. **Verifying Jordan Forms and Numerical Stability.** Let $T_A : \mathbb{R}^4 \to \mathbb{R}^4$ be the transformation defined by $T_A(v) = Av$ where

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 11 & 6 & -4 & -4 \\ 22 & 15 & -8 & -9 \\ -3 & -2 & 1 & 2 \end{pmatrix}.$$

(a) Compute the Jordan canonical form of T_A by computing the minimal polynomial of T_A. Verify your result in Matlab and use Matlab to find a Jordan basis for T_A.

(b) Now consider the matrix obtained by adding a small random matrix to A; in Matlab use the command

$$B = A + \text{randn}(4)/100.$$

This might be the matrix you would observe if you were trying to measure A in a noisy environment. Use Matlab to compute the Jordan form of B.

(i) Does B have the same Jordan form as A?

(ii) If not, are the eigenvalues of B and A close to each other?