ADE singularities and Hirota Quadratic Equations

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The Bosonic Fock space, by definition, is the infinite dimensional vector space underlying the polynomial algebra $B = \mathbb{C}[x_1, x_2, \ldots]$. Given a vector $\tau \in B$, the coordinates of τ with respect to the monomial basis $\{x_1^{i_1} x_2^{i_2} \ldots\}$ are called Fourier coefficients of τ.

In 1981, M. Sato, noticed that the solutions of the KP-hierarchy (which is a certain infinite sequence of commuting flows on the loop space $\mathcal{L}(\mathbb{C}^\infty)$) can be parameterized by vectors in the Fock space B, such that their Fourier coefficients satisfy certain quadratic relations. In fact, these relations are the Plücker relations describing the embedding of a certain infinite dimensional Grassmanian in B. After Sato's work, for many other hierarchies it was proven that their solutions can be parameterize by vectors in B, called *tau-functions* whose Fourier coefficients satisfy quadratic relations, called *Hirota Quadratic Equations* (shortly HQE).

In this talk, I am planning to explain how to construct HQE in the settings of singularity theory. More precisely, for each holomorphic function in $\mathbb{C}^{n+1}$ with an isolated critical point at 0 of A, D, or E type, using the theory of vanishing cycles and period mappings, we can prove that *the total ancestor potential* of f satisfies certain HQE. The total ancestor potential of f is an analogue of the generating function of ancestor Gromov–Witten invariants of a compact Kähler manifold X. The far reaching goal of our project is to construct HQE for the different generating functions in GW theory i.e., to derive quadratic relations for the GW invariants of the manifold X.