Threshold models for rainfall and convection: deterministic and stochastic triggers

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Motivation

- Modeling precipitation in the tropics.
- Global Climate Models (GCM) capture mean statistics well.
- GCMs have relatively small variability compared to observations.

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- Can a choice in trigger to convection increase variability?
Goals for this Study

- **Goals:**
  1. Model moisture, in a single column, with a trigger that indicates precipitation (strong convection).
  2. Introduce 4 simple models of column water vapor with different triggers and thresholds.
  3. Compare and contrast the models with aim of producing statistics that mirror observational data and increasing variability.
  4. Show that the model statistics are “close” to one another for large transition rates.
\[ \frac{dq_t}{dt} = \begin{cases} 
    m + D_0 \dot{W}_t, & \sigma_t = 0 \\
    -r + D_1 \dot{W}_t, & \sigma_t = 1 
\end{cases} 
\]

- \( D_1 > D_0 > 0 \)

**Deterministic Trigger**

**Stochastic Trigger**

**One Threshold (D1)**
- In wet state
- Switch to wet
- Switch to dry
- In dry state

**Two Thresholds (D2)**
- In wet state
- No change in dynamics
- Switch to wet
- Switch to dry
- In dry state

**S1**
- Switch to wet state at random time
- Switch to dry state at random time

**S2**
- Switch to wet state at random time
- No change in dynamics
- Switch to dry state at random time

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Threshold Models

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Mean and Variance of Precip.

- Stochastic trigger and (D2) capture **pick up** in the mean and **spike** in variance.

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A Robust Statistic

- Average Rainfall.
  - In steady state,
    
    $$E[r\sigma] = \frac{r^2}{m+r}.$$  
    
    Independent of \# of thresholds and stochastic vs deterministic trigger.

- Replacing a deterministic trigger with a stochastic one improves variability while preserving the mean.
The (D2) model has simpler formulas.

Question: are we justified in using (D2) to approximate (S2)?

- (D2) becomes (D1) for $q^c - q^{np} = 0$.
- Same with (S2) to (S1).
- Q: What about (D2) to (S2)?
Convergence of Models

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- A: **Theorem:** (S2) converges (in \( L^2 \)) to (D2) as the transition rate tends to infinity.
Two types of Error. 1) $\xi$ from delay in jumping time. 2) $\zeta$ accruing “catch up” error.
Conclusions

- We carried out a systematic study of the differences of stochastic/deterministic trigger, and one/two thresholds.
- The stochastic trigger allows better variability while preserving average rainfall.
- The two threshold models capture relevant observational statistics.
- While (S2) captures the statistics more accurately, the exact formulas are complex and in some cases, very hard to analyze.
- (D2) approximates (S2) with large transition rates.