First name:__________________

Last name:__________________

Student ID:__________________

TA and (if known) section #:__________________

There are 5 problems, and a total of 8 pages. Instructions:

- Penalties for cheating are described in the class syllabus, and may include failure in the course. The only items that may be on your desk are a writing implement, an eraser, and the exam. No pencil bags, and backpacks must be closed and cellphones turned off and put away.
- Do not open the book until you are told to do so.
- A correct solution is one that is justified by the work shown. Little to no credit will be given for mysterious or unsupported answers.
- Write legibly, and clearly indicate what should be graded and where it can be found.
- All pages of the exam must be returned.
1. a. Find an equation for the plane containing the lines

\[
\begin{align*}
  x &= 1, \\
  y &= 3 + t, \\
  z &= 1 - t
\end{align*}
\quad \text{and} \quad
\begin{align*}
  x &= t, \\
  y &= 5 - 2t, \\
  z &= 1.
\end{align*}
\]

b. Find parametric equations for the line through the origin that is perpendicular to the plane in a.
2. An ant travels from \((1, 0, 1)\) to \((-e^\pi, 0, e^\pi)\) along the curve parametrized by the vector function

\[
\vec{r}(t) = \langle e^t \cos t, e^t \sin t, e^t \rangle,
\]

but the ant walks with a constant speed of 2. How much time does the ant’s journey take?
3. A particle moves with acceleration

$$\vec{a}(t) = \langle 1, t, t^2 \rangle.$$

If the particle passes through the origin with velocity $$\langle 1, 1, 1 \rangle$$ at time $$t = 1$$, find the particle’s position at time $$t$$. 
4. Consider the function \( f(x, y) = x^2 + 4y^2 \).

a. Sketch the level sets of this function at levels \(-5, 0, \text{ and } 5\).

b. Sketch the 3D graph of this function. Label your axes, and make sure the intersections with the coordinate planes appear in your sketch.
5. Let $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ be a smooth parametrization, that is, $\vec{r}'(t)$ always exists and is never the zero vector. Use the basic differentiation rules (product rule, chain rule) to show that

$$\vec{T}'(t) = \frac{\vec{r}''(t)}{\|\vec{r}'(t)\|} - \frac{\vec{r}'(t) \cdot \vec{r}''(t)}{\|\vec{r}'(t)\|^3} \vec{r}'(t),$$

where $\vec{T}(t)$ denotes the unit tangent vector. (If you get stuck, for partial credit, you may show that the formula works for the curve $\vec{r}(t) = \langle 1, t, t^2 \rangle$.)
Extra extra paper A
Extra extra paper B