MATH 234 (STOVALL). HOMEWORK 3. DUE WEDNESDAY, 9/27.

1. The position vector of a particle in the plane is given by the vector function \( \vec{r}(t) = (t + 1)\vec{i} + (t^2 - 1)\vec{j} \). Sketch the particle’s path, and at time \( t = 1 \), find the velocity, speed, and acceleration of the particle, as well as the equation for the tangent line.

2. Find an equation for the tangent line to the curve \( \vec{r}(t) = (\cos t - t^3)\vec{i} + (\sin t + t)\vec{j} + e^t\vec{k} \) at time \( t = 0 \).

3. Show that if \( \vec{a}(t), \vec{b}(t), \text{ and } \vec{c}(t) \) are differentiable vector functions, then
\[
\frac{d}{dt} [\vec{a} \cdot (\vec{b} \times \vec{c})] = \vec{a}' \cdot (\vec{b} \times \vec{c}) + \vec{a} \cdot (\vec{b}' \times \vec{c}) + \vec{a} \cdot (\vec{b} \times \vec{c}').
\]
Check this by direct computation in the case \( \vec{a}(t) = (\sin t, \cos t, 0), \vec{b}(t) = (-\cos t, \sin t, 0), \vec{c}(t) = (0, 0, t) \).

4. Problem 4 in the Calculus notes (Chapter 2, Section 17).

5. Problem 5 in the Calculus notes (Chapter 2, Section 17).

6. Find the position vector as a function of time of a particle whose acceleration at each time \( t \) is \( \langle 0, 2, 6t \rangle \), who passes through the origin at time \( t = 1 \), and whose velocity at time \( t = 2 \) is \( \vec{0} \).

7. Find speed and the unit tangent vector to the curve
\[
\vec{r}(t) = t \sin t \vec{i} - t \cos t \vec{j} + \frac{2\sqrt{2}}{3} t^{\frac{3}{2}} \vec{k}
\]
as a function of time. Find the length of the portion of the curve corresponding to \( 0 \leq t \leq \pi \).

8. An ant walks around the circle of radius 2 centered at the point \( (1, 1) \).
   i. Find some parametrization for this circle. What is the speed of your parametrization?
   ii. If the ant leaves the point \( (3, 1) \) at time \( t = 0 \) and walks counterclockwise at a constant speed of 3, what is the ant’s position at time \( t \)? How long does it take the ant to make one trip around the circle?
   iii. If the ant leaves the point \( (3, 1) \) at time \( t = 0 \) and walks counterclockwise, but now the ant’s speed at each time \( t \) is equal to \( t \), what is the ant’s position vector as a function of time?

9. A (very narrow) ramp follows the helix \( \vec{r}(t) = (\cos t, \sin t, t) \). An ant is walking up this ramp, and at the point \( (0, 1, \frac{\pi}{2}) \), the ant’s speed is 6.
   i. What is the ant’s velocity at the point \( (0, 1, \frac{\pi}{2}) \)?
   ii. What is the ant’s velocity at \( (0, 1, \frac{\pi}{2}) \) if the ant has speed 6 at this point, but is walking down the ramp instead of up?