MATH 234 (STOVALL). HOMEWORK 5. DUE WEDNESDAY, 10/18.

1. Show that the following functions are not continuous at (0,0) by finding a continuous path along which they are not continuous. Use a computer graphing program to see what the graph of the function looks like near the discontinuity.
   a. \( f(x, y) = \frac{x}{\sqrt{x^2 + y^2}} \), for \((x, y) \neq (0, 0)\), \( f(0, 0) = 0 \).
   b. \( g(x, y) = \frac{x}{x^2 + y^2} \), for \((x, y) \neq (0, 0)\), \( f(0, 0) = 0 \).
   c. \( h(x, y) = \frac{xy}{x^2 + y^2} \), for \((x, y) \neq (0, 0)\), \( f(0, 0) = 0 \).
   d. \( p(x, y) = \frac{x^2 - y^2}{x^2 + y^2} \), for \((x, y) \neq (0, 0)\), \( f(0, 0) = 0 \).
   e. \( q(x, y, z) = \frac{2xy + yz}{x^2 + z^2} \), for \((x, z) \neq (0, 0)\), \( f(x, y, z) = 0 \), for \(x = z = 0\).

2. Problem 2 in Chapter 4, Section 3 (pg 51-2) of the notes.

3. Let \( r(x, y) = \sqrt{x^2 + y^2} \) be the polar radius function and \( \theta(x, y) \) be the polar angle function, \( \theta(x, y) = \arctan(\frac{y}{x}) \) in the right half-plane and \( \theta(x, y) = \arctan(\frac{y}{x}) + \pi \) in the left half-plane.
   a. Compute the partial derivatives of \( r \) and show that they can be written as
      \[ \frac{\partial r}{\partial x} = \frac{x}{r}, \quad \frac{\partial r}{\partial y} = \frac{y}{r}. \]
   b. Use the \( \arctan \) formula to compute the partial derivatives of \( \theta \). Show that
      \[ \frac{\partial \theta}{\partial x} = -\frac{\sin \theta}{r}, \quad \frac{\partial \theta}{\partial y} = \frac{\cos \theta}{r}. \]

4. Problem 6 in Chapter 4, Section 3 (pg 51-2) of the notes.

5. Problem 7 in Chapter 4, Section 3 (pg 51-2) of the notes.

6. Problem 1 in Chapter 4, Section 7 (pg 61) of the notes.

7. Use implicit differentiation and the formulas
   \[ x = r \cos \theta, \quad y = r \sin \theta \]
to re-derive the formulas in parts a and b of Problem 3.

8. Compute \( \frac{\partial z}{\partial u} \) and \( \frac{\partial z}{\partial v} \) at the given point in two ways. First, use the Chain rule, and, second, express \( z \) as a function of \( u \) and \( v \) and differentiate directly:
   a. \( z = e^x \ln y, \quad x = \ln(u \sin v), \quad y = e^{u \cos v}, \quad (u, v) = (\ln 2, \frac{\pi}{2}) \).
   b. \( w = \ln(x^2 + y^2 + z^2), \quad x = uv \sin u, \quad y = uv \cos u, \quad z = uv, \quad (u, v) = (-2, 0) \).