
For problems 1-2, a) Compute the gradient of the given function at the given point; b) Write the equation of the level set of the function through the point; c) Find a unit vector tangent to the level set in part b, a unit vector perpendicular to that level set, the direction of fastest increase of the function at the given point, and the direction of fastest decrease; d) Compute the directional derivative in each of the directions in part c.

1. \( f(x, y) = \tan^{-1}\left(\frac{y}{\sqrt{x}}\right) \), at \((1, 1)\).

2. \( f(x, y, z) = (x^2 + y^2 + z^2)^{-\frac{1}{2}} + e^{xyz} \), at \((0, 3, 4)\).

For problems 3-4, find equations for the plane tangent to and the line normal to the given surface at the given point.

3. The graph of \( f(x, y) = (1 - x + xy)^2 \) at \((0, 0, 1)\).

4. The surface \( \sin(\pi x) + e^{xyz} = x^2y \), at \((\frac{3}{2}, 0, 4)\).

5. Find parametric equations for the tangent line to the curve of intersection of the surfaces \( x^2 + y^2 = 4 \), \( x^2 + y^2 - z = 0 \), at the point \((2, 0, 4)\).

6. Find the linear approximation to the function \( f(x, y, z) = e^{xyz} \sin(x^2 + y^2 + z^2) \) near the points a) \((0, 0, 1)\) and b) \((\ln 2, 1, 1)\).

7. Find all critical points of the functions below.
   a. \( f(x, y) = x^2 + 4xy + y^2 - 6y + 1 \)
   b. \( f(x, y) = xy(4 - x - 2y) \)
   c. \( f(x, y) = x^2y \)
   d. \( f(x, y, z) = \ln(x^2 + y^2 + z^2 + 1) - x \)

For 8-10, find the absolute maximum and minimum values of the given function on the given region:

8. \( f(x, y) = (x - x^2) \sin y \) on the rectangular region bounded by \(-1 \leq x \leq 1\) and \(0 \leq y \leq \pi\).

9. \( f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8 \) on the triangular region bounded by the lines \( x = 0 \), \( y = 0 \), and \( x + y = 1 \).

10. \( f(x, y) = ax + by \) on the region \( x^2 + y^2 \leq 1 \).