There are 6 problems, and a total of 9 pages. If you use the extra pages at the end, clearly indicate that you have done so, and clearly label the extra pages with the corresponding problem number.

**Instructions:** Solve each problem in the space provided, completely and carefully justifying each deduction. Unless otherwise stated, you can use any result that we proved in class. You may **not** consult any books, notes, peers, internets, or other outside resources. Cheating will not be tolerated.

**Grading notes:**

- Problems will be graded on correctness and completeness of the solution, but also on style (so write in clear, complete sentences).

- Partial credit will be given for correct definitions of the relevant terms, statements of relevant theorems, helpful pictures, or other positive progress. If you get stuck, tell me what you do know (within reason).

- Credit is **not** based on the length of your answer, so write proofs, not essays.
1. Let \((t_n)\) and \((s_n)\) be two sequences in \(\mathbb{C}\), and assume that \(\lim s_n = s\) and \(\lim t_n = t\). Prove that \(\lim(s_n - t_n) = s - t\). (Everything is with respect to the absolute value metric on \(\mathbb{C}\).)
2. Here the space is $\mathbb{R}$ with the Euclidean metric.

a) Exhibit an open cover of $E := \{ \frac{1}{n^2} : n \in \mathbb{N} \}$ that contains no finite subcover of $E$. (You do not need to prove that it does not have a finite subcover.)

b) Prove that $\left\{ \frac{1}{n^2} : n \in \mathbb{N} \right\} \cup \{0\}$ is compact. (If you use a certain theorem, you must prove that its hypotheses are satisfied.)
3. Let $(X,d)$ be a metric space and let $F_1$ and $F_2$ be two closed subsets of $X$. Use the definition to prove that $F_1 \cup F_2$ is closed.
4. Use the definition to prove that
\[
\frac{n^3 + (-1)^n 3n^2 + 1}{n^3 + 2n^2 + (-1)^n}
\]
converges.
5. Let $X$ be any metric space. Let $p_1$ and $p_2$ in $X$ and define

$$G = \{ q \in X : d(p_1, q) < d(p_2, q) \}$$

Prove that $G$ is open.
6. Part 1: Give the precise, detailed **negation** of the definition of the following

a) The subset $E$ of the metric space $(X, d)$ is connected.

b) The sequence $(s_n)$ in $\mathbb{R}$ is monotonic (same as monotone).

c) The sequence $(p_n)$ in the metric space $(X, d)$ is convergent.

d) The sequence $(p_n)$ in the metric space $(X, d)$ is Cauchy.

e) The point $p$ is an interior point of the set $E$.

Remember that you are giving the **negations** above.
Extra page