1. Consider the linear operator $T : \mathbb{R}^{N+1} \to C^N([0, 1])$ defined by $T(a_0, \ldots, a_N)(x) = \sum_{j=0}^N \frac{1}{j!} a_j x^j$. Show this is a bounded linear operator and find its norm.

2. Consider the mapping $\Psi : C^0([0, 1]) \to C^1([0, 1])$ defined by $\Psi[f](x) = \int_0^x f(t)^4 \, dt$, for $0 \leq x \leq 1$.
   a. Show that $\Psi[f]$ is actually in $C^1([0, 1])$ for each $f \in C^0([0, 1])$.
   b. For each $f \in C^0([0, 1])$, find a linear mapping $L_f : C^0([0, 1]) \to C^1([0, 1])$, and another mapping $E_f : C^0([0, 1]) \to C^1([0, 1])$ so that for all $h \in C^0([0, 1])$,
      \[ \Psi[f + h] = \Psi[f] + L_f[h] + E_f[h], \]
      and $\lim_{h \to 0} \frac{\|E_f[h]\|_{C^1}}{\|h\|_{C^0}} = 0$.
   c. Show that $L_f$ is a bounded operator for each $f$ (hence $L_f = D\Psi_f$, the derivative of $\Psi$ at $f$), and find its operator norm.