1. Show that the rank theorem implies the implicit function theorem.

2. Show that the equations
   \[ x^2 - y^2 - u^3 + v^2 = -4 \]
   \[ 2xy + y^2 - 2u^2 + 3v^4 = -8 \]
determine functions \( u(x, y), v(x, y) \) near \( x = 2, y = -1 \) such that \( u(2, -1) = 2, v(2, -1) = 1 \). Compute \( \frac{\partial u}{\partial x} \).

3. Determine whether the “curve” described by the equation \( x^2 + y + \sin(xy) = 0 \) can be written in the form \( y = f(x) \) in a neighborhood of \((0, 0)\). Does the implicit function theorem allow you to say whether the equation can be written in the form \( x = h(y) \) in a neighborhood of \((0, 0)\)?

4. Let \( g_1, \ldots, g_k, f : \mathbb{R}^n \rightarrow \mathbb{R} \) be continuously differentiable. If \( \nabla g_1, \ldots, \nabla g_k \) are linearly independent at \( x_0 \) and \( \nabla g_1, \ldots, \nabla g_k, \nabla f \) are linearly dependent on a neighborhood of \( x_0 \), then there exists a neighborhood \( U \) of \( x_0 \), a neighborhood \( V \subseteq \mathbb{R}^k \) of \( (g_1(x_0), \ldots, g_k(x_0)) \), and a continuously differentiable function \( F : V \rightarrow \mathbb{R} \) such that on \( U \), \( f(x) = F(g_1(x), \ldots, g_k(x)) \).