PROBLEMS (Turn these in. A subset will be graded.)

1. Show by example that if $X$ is not compact, a uniformly bounded equicontinuous sequence in $C^0(X)$ need not have a uniformly convergent subsequence. (Recall that $F \subseteq C^0(X)$ is uniformly bounded if there exists $M$ independent of $f$ such that $|f(x)| \leq M$ for all $x \in X$ and $f \in F$.)

2. Prove that if $(f_n)$ is a bounded sequence in $C^1([0,1])$, then $(f_n)$ has a subsequence that converges uniformly (i.e. in $C^0([0,1])$).

3. Problem 15 in Rudin.

4. Problem 16 in Rudin.

5. Problem 18 in Rudin.

6. Problem 20 in Rudin.

7. Let $f \in C^k([a,b])$. For all $\varepsilon > 0$, there exists a polynomial $P$ such that $\|f - P\|_{C^k([a,b])} < \varepsilon$. (Hint: Use some tools.)

EXERCISES (do not turn these in)

Read Problem 17 in Rudin and think about why this makes sense.
Honors Problems (Keep these for now.)

Prove that if \((f_n)\) is a pointwise bounded equicontinuous sequence in \(C(\mathbb{R})\), then \((f_n)\) has a locally uniformly convergent subsequence.

If \(f \in C^0([a, b])\) is not a polynomial, and \((P_n)\) is a sequence of polynomials that converge uniformly to \(f\) on \([a, b]\), then the degrees of the \(P_n\) must diverge to infinity.