MATH 522 (STOVALL). HOMEWORK 5. DUE WEDNESDAY, 10/22

Problems (Turn these in. A subset will be graded.)

1. Problem 9 of Chapter 3 in Rudin.

2. Problem 10 of Chapter 3 in Rudin.

3. Problem 1 of Chapter 8 in Rudin.

4. Problem 2 of Chapter 8 in Rudin.

5. Problem 3 of Chapter 8 in Rudin.

6. Assume that $f(x) = \sum_{n=0}^{\infty} b_n x^n$, with convergence in $|x| < R$. Assume that $f(0) \neq 0$. Then it is a fact that $\frac{1}{f}$ has a power series expansion about $x = 0$. Use the fact that $f \cdot \frac{1}{f} = 1$ and Cauchy products to find (and prove!) a formula for the coefficients of the power series for $\frac{1}{f}$ in terms of the $b_n$.

An alternative derivation of this formula comes from the computation

$$\frac{1}{f(x)} = \frac{1}{f(0)(1 + \frac{f(x)-f(0)}{f(0)})} = \sum_{n=0}^{\infty} \frac{1}{f(0)} (-1)^n \left( \frac{f(x)-f(0)}{f(0)} \right)^n,$$

and the use of Cauchy products to rewrite the right hand side as a power series. (You do not have to carry out this derivation.)

Exercises (do not turn these in)

Prove that power series expansions are unique. In other words, if $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$, $|x-a| < R_1$ and $f(x) = \sum_{n=0}^{\infty} d_n(x-a)^n$, $|x-a| < R_2$, with $R_1, R_2 > 0$, then $c_n = d_n$ for all $n$. 
Honors Problems (Keep these for now.)

Prove that if $f$ is analytic on the open interval $I$ and $g$ is analytic on the open interval $J$, and $g(J) \subseteq I$, then $f \circ g$ is analytic on $J$. 