There are 3 problems, and a total of 7 pages.

**Instructions:** Solve each problem in the space provided, completely and carefully justifying each deduction. You may **not** consult any books, notes, peers, internets, or other outside resources. Cheating will not be tolerated.

**Suggestions:**

- Each problem is worth 10 points.
- Problems will be graded on correctness and completeness of the solution, but also on style (so write in clear, complete, readable sentences).
- Partial credit will be given for correct definitions of the relevant terms, statements of relevant theorems, helpful pictures, or other positive progress. If you get stuck, tell me what you do know (within reason).
- Credit is **not** based on the length of your answer, so write proofs, not essays.

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1. Let \((X, d)\) be a metric space and let \(F \subseteq C^0(X)\) be a collection of functions.

   a. (3 pts) State, precisely, in the simplest terms possible, what it means for \(F\) to be each of the following: totally bounded, pointwise bounded, equicontinuous.

   b. (1 pt) State the Arzela–Ascoli theorem.

   c. (6 pts) Prove that if \(F\) is totally bounded, then \(F\) is pointwise bounded.
Extra paper for problem 1.
2. a. (7 pts) Let \((X, d)\) be a metric space. Assume that the sequence \((f_n)\) in \(C^0(X)\) converges uniformly to some function \(f\). Let \(x \in X\) and assume that \(x_n \to x\). Prove that

\[
\lim_{n \to \infty} f_n(x_n) = f(x).
\]  

(1)

b. (3 pts) If (1) holds for every convergent sequence \((x_n)\) (where \(x\) denotes the limit), does it follow that \(f_n \to f\) uniformly? Proof or counterexample.
Extra paper for problem 2.

b. (8 pts) Let \((X, d)\) be a compact metric space and let \(x_0 \in X\). Prove that the closed unit ball \(B := B_1(x_0)\) is not contained in a countable union of closed sets with empty interior.
Extra paper for problem 3.