1. Let $X$, $Y$ be normed vector spaces. Prove that if the function $T : X \to Y$ is a bounded linear transformation, then its graph, $\Gamma(T) := \{(x, Tx) : x \in X\}$ is a closed linear subspace of $X \times Y$.

2. Let $X$ and $Y$ be Banach spaces and let $T \in \mathcal{L}(X, Y)$. Then there exists a constant $C > 0$ such that $\|Tx\| \geq C\|x\|$ for every $x \in X$ if and only if $T$ is one-to-one and $TX$ is a (topologically) closed (vector) subspace of $Y$.

3. Let $X$ and $Y$ be a Banach spaces, and let $\Omega(X, Y)$ be the space of all bounded linear operators from $X$ to $Y$, whose inverses also exist and are bounded linear operators from $Y$ to $X$. Prove that the inversion operator $\text{Inv} : \Omega(X, Y) \to \Omega(Y, X)$ is differentiable, and that $D \text{Inv}_A(h) = -A^{-1}hA^{-1}$, $h \in \mathcal{L}(X, Y)$.

Hint: For $h$ sufficiently small, $(A + h)^{-1}$ may be computed using the von Neumann series:

$$
(A + h)^{-1} = \sum_{n=0}^{\infty} (-1)^n (A^{-1}h)^n A^{-1} = \sum_{n=0}^{\infty} (-1)^n A^{-1}(hA^{-1})^n.
$$

4. If $f : V \to \mathbb{R}$ is differentiable at $v \in V$ and $f$ has a local maximum at $v$, then $Df_v = 0$. 