Problems

1. Problem 36 in Chapter 5 of Folland
2. Problem 40 in Chapter 5 of Folland
3. Problem 48 in Chapter 5 of Folland
4. Problem 53 in Chapter 5 of Folland
5. Problem 57 in Chapter 5 of Folland
6. Problem 63 in Chapter 5 of Folland
7. There does not exist a norm consistent with the Fréchet topology induced by the semi-norms $\rho_{\alpha\beta}(f) = \sup_{x \in \mathbb{R}^d} |x^\alpha \partial^\beta f(x)|$, where $\alpha, \beta$ range over all multi-indices in $\mathbb{Z}_d_{\geq 0}$.
8. Let $f, g \in L^p(\mu)$ and $1 < p < \infty$. Prove that $\|f + g\|_p = \|f\|_p + \|g\|_p$ if and only if $f = 0$ or $g$ is a positive scalar multiple of $f$. Prove that $p \neq 1, \infty$ is necessary.
9. Let $f \in L^\infty(\mu)$ and assume that $f \in L^{p_0}(\mu)$ for some $p_0 < \infty$. Prove that $\|f\|_\infty = \lim_{p \to \infty} \|f\|_p$.
10. For each $1 \leq p \leq \infty$, construct an $L^p(\mathbb{R})$ function that does not lie in $L^q(\mathbb{R})$ for any other value of $q$.
11. Problem 4 in Chapter 6 of Folland