Some of these are from Stein–Shakarchi, Book IV, Functional Analysis.

1. Problem 34 on page 308 of Folland. The space Folland denotes by $\Lambda_\alpha(\mathbb{R}^d)$ is (perhaps more) commonly called the Hölder class $C^{0,\alpha}(\mathbb{R}^d)$.

2. The Fourier series of a generic function in $C(\mathbb{T})$ diverges on a generic subset of $\mathbb{T}$. Hint: Choose a dense sequence $\{\theta_i\} \subseteq \mathbb{T}$. Let
   \[ G_i := \{ f \in C(\mathbb{T}) : \{S_N f\} \text{ diverges at } x_i \}. \]
   Show $G := \cap_i G_i$ is generic. Let $f \in G$. Show $E_n := \{ \theta : \sup_N |S_N f(\theta)| > n \}$ is open and dense.

3. Let $X, Y$ be Banach spaces and $T : X \to Y$ a bounded linear operator. Either $T$ is surjective, or its image is of the first category.

4. For $n \geq 2$, let $\Lambda_n$ denote the set of real numbers $x$ such that there exist infinitely many distinct fractions $\frac{p}{q}$ with $|x - \frac{p}{q}| \leq \frac{1}{q^n}$. Show that $\Lambda_n$ is a measure zero generic subset of $\mathbb{R}$.

5. The metric
   \[ \rho(f, g) := \sum_{n=0}^{\infty} 2^{-n} \frac{\rho_n(f - g)}{1 + \rho_n(f - g)}, \]
   where $\rho_n(h) := \|h^{(n)}\|_{C^0([0,1])}$, makes $C^\infty([0,1])$ into a complete metric space. (You do not need to show this.) The smooth function $f$ is analytic on a neighborhood of $x_0$ if the Taylor series of $f$ at $x_0$ converges to $f$ on a neighborhood of $x_0$, and singular at $x_0$ if it is analytic on no neighborhood of $x_0$. Prove that the set of $C^\infty([0,1])$ functions that are singular at every point is generic.
   Hint: Let $F_\varepsilon$ denote the set of $C^\infty([0,1])$ functions $f$ for which the Taylor series at some $x^* \in [0,1]$ has radius of convergence at most $\varepsilon$. Show $F_\varepsilon$ is closed and nowhere dense.