<table>
<thead>
<tr>
<th>Problem</th>
<th>Ans</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>(a)</td>
<td></td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Tree Diagram" /></td>
<td></td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Event E" /></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( { H_A, H_K, H_Q, T_K } )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( P_r(E) = \frac{4}{27} + \frac{3}{27} + \frac{2}{27} + \frac{6}{27} = \frac{15}{27} = \frac{5}{9} )</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3/10</td>
<td></td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Tree Diagram" /></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( P_r{ \text{dime selected} } )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = P_r{ 10, 20 } = \frac{2}{10} + \frac{1}{10} = \frac{3}{10} )</td>
<td></td>
</tr>
</tbody>
</table>
Problem 5

$\text{Ans}$

Reason

- $E = \{TTH, THH\}$
- $P_r(E) = \frac{1}{8}$
Problem | Ans | Reason
--- | --- | ---
7 | (a) $\frac{5}{7}$ | 
(b) $\frac{5}{6}$ | 
Pr $\left[ \text{Second is red} \right] = \frac{5}{9} \cdot \frac{9}{8} + \frac{5}{9} \cdot \frac{8}{9} = \frac{45}{72} + \frac{40}{72} = \frac{85}{72} = \frac{9}{8}$  

Pr $\left[ \text{at least one red} \right] = \frac{5}{9} \cdot \frac{4}{8} + \frac{5}{9} \cdot \frac{4}{8} + \frac{4}{9} \cdot \frac{5}{8} = \frac{60}{72} = \frac{5}{6}$

8 | $\frac{17}{50}$ | 

Pr $\left[ \text{Second bull Red} \right] = \frac{2}{5} \cdot \frac{1}{9} + \frac{3}{5} \cdot \frac{2}{5}$  

$= \frac{2}{5} \left( \frac{1}{9} + \frac{3}{5} \right) = \frac{2}{5} \cdot \frac{17}{20}$  

$= \frac{17}{50} = \frac{17}{50}$
<table>
<thead>
<tr>
<th>Problem</th>
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<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>$\frac{19}{30}$</td>
<td></td>
</tr>
</tbody>
</table>

\[
P_r \left[ \text{selected course is in afternoon} \right] = \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{4}{6} = \frac{1}{2} \left( \frac{3}{5} + \frac{4}{6} \right) = \frac{1}{2} \times \frac{19}{15} = \frac{19}{30}
\]

| 10      | .28   |        |

\[
P_r \left[ \text{outcome of state 2 is a} \right] = (.4)(.4) + (.6)(.2) = .16 + .12 = .28
\]
Problem | Ans | Reason
--- | --- | ---
11 | (a) 0.67 |
(b) 0.845 |

\[
\Pr[\text{car has CO}] = (0.20)(0.90) + (0.45)(0.70) + (0.35)(0.50)
\]
\[
= 0.18 + 0.315 + 0.175
\]
\[
= 0.67
\]

\[
\Pr[\text{car has CO}] = (0.20)(0.90) + (0.45)(0.70) + 0.35
\]
\[
= 0.180 + 0.315 + 0.350
\]
\[
= 0.845
\]

12 | 0.36 |

\[
\Pr[\text{Joe late}] = (0.5)(0.3) + (0.7)(0.4) + (0.1)(0.5)
\]
\[
= 0.15 + 0.28 + 0.05
\]
\[
= 0.48
\]
Problem | Ans | Reason
--- | --- | ---
13 | (a) $4/7$ | 

(b) $3/5$ |

\[
\Pr \left[ \text{exactly one blue} \right] = \frac{4}{7} \cdot \frac{3}{6} + \frac{3}{7} \cdot \frac{4}{6} = \frac{4}{7}
\]

\[
\Pr \left[ \text{2nd ball blue} \mid \text{at least one blue} \right] = \frac{\Pr \left[ \text{2nd ball blue} \right]}{\Pr \left[ \text{at least one blue} \right]}
\]

\[
\Pr \left[ \text{2nd ball blue} \right] = \frac{4}{7} \cdot \frac{3}{6} + \frac{3}{7} \cdot \frac{2}{6} = \frac{18}{42}
\]

\[
\Pr \left[ \text{at least one blue} \right] = \frac{4}{7} \cdot \frac{3}{6} + \frac{3}{7} \cdot \frac{2}{6} + \frac{3}{7} \cdot \frac{4}{6} = \frac{30}{42}
\]

\[
\text{Ans} = \frac{18}{42} \cdot \frac{42}{30} = \frac{18}{30} = \frac{6 \cdot 3}{6 \cdot 5} = \frac{3}{5}
\]

14 | 0.39 | 

\[
\Pr \left[ \text{student is NR} \right] = (0.4)(0.3) + (0.6)(0.95)
\]

\[
= 0.12 + 0.57
\]

\[
= 0.39
\]
Problem 15

\[ \Pr \left[ \text{on time} \right] = (0.5 \times 0.8) + (0.2 \times 0.6) + (0.3 \times 0.4) = 0.40 + 0.12 + 0.12 = 0.64 \]

\[ \Pr[EF] = \frac{3}{5} \]

\[ \Pr[EF] = \frac{1}{5} \]

\[ F/E \quad F'/E \]
\[ \frac{1}{2} \quad \frac{1}{2} \]
\[ E'F \quad E'F' \]
\[ \frac{3}{5} \quad \frac{2}{5} \quad x = \frac{1}{3} \]

\[ \Pr[F] = \Pr[\{EE, E'F\}] = \frac{2}{5} \times 0.5 + \frac{3}{5} \times \frac{2}{3} = \frac{3}{5} \]
\[ \frac{1}{2} = \Pr(F | E) = \frac{\Pr(E \cap F)}{\Pr(E)} \]

\[ = \frac{y}{\frac{2}{15}} = \frac{5}{2} y \]

\[ y = \frac{1}{5} \text{ so } \Pr(E \cap F) = \frac{1}{5} \]

\[ \Pr(F) = y + \frac{2}{5} = \frac{1}{5} + \frac{2}{5} = \frac{3}{5} \]

\[
\begin{align*}
1 - \frac{1}{2} - \frac{1}{6} &= \frac{1}{2} \\
\frac{3}{2} \quad \frac{1}{4} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{1}{3} \\
\frac{3}{8} \quad \frac{1}{8} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{2}{9} \quad \frac{1}{9}
\end{align*}
\]
$$P_{T}(E) = (0.2)(0.2)^{4} + (0.2)(0.8)^{4} + (0.2)(0.8)^{4} + (0.2)(0.8)^{4} + 2(0.8)^{4} = 0.096$$
Problem 20

$\Pr[\text{test pos}] = (0.02)(0.9) + (0.98)(0.1) = 0.018 + 0.098 = 0.116$

Problem 21

$\Pr[RR] = \frac{1}{3}$

$\Pr[RR | 1\text{st ball } R] = \frac{1}{2}$

$\Pr[2 \text{ red balls}] = \frac{1}{3}$

$\Pr[2 \text{ red } | \text{ 1st ball red}] = \frac{1}{2}$
Problem 22

\[ P \]  
\[ \begin{array}{c}
\text{Reason for purchase} \\
\text{Satisfied?} \\
1.35 \\
0.15 \\
0.15 \\
0.15 \\
0.15 \\
\end{array} \]

\[ P \left[ \text{satisfied} \right] = 0.175 + 0.4 + 0.045 = 0.62 \]

Problem 23

\[ \frac{13}{24} \]

\[ \begin{array}{c}
\text{Drawer} \\
\text{Coins} \\
QQ \, QN \\
QQ \, NN \\
\end{array} \]

\[ P_r (E) = \frac{5}{9} \cdot \frac{3}{8} + \frac{3}{9} \cdot \frac{5}{8} + \frac{3}{9} \cdot \frac{1}{8} + \frac{3}{9} \cdot \frac{2}{8} \]

\[ = \frac{5}{9} \cdot \frac{3}{8} + \frac{3}{9} = \frac{1}{3} \left( 1 + \frac{5}{8} \right) \]

\[ = \frac{1}{3} \cdot \frac{13}{8} = \frac{13}{24} \]
<table>
<thead>
<tr>
<th>Problem</th>
<th>Ans</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 24</td>
<td></td>
<td><img src="image" alt="Tree Diagram" /></td>
</tr>
<tr>
<td>(b) 2/3</td>
<td></td>
<td>$P_r\left[ r \text{ ball drawn} \right] = \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{2}{4} = \frac{3}{8}$</td>
</tr>
</tbody>
</table>

<p>| 25 | 0.272 | <img src="image" alt="Tree Diagram" /> |
|    |     | $P_r(\text{E}) = (0.2)(0.6)(0.4) + (0.2)(0.7)(0.4) + (0.8)(0.4)(0.6) = 0.272$ |</p>
<table>
<thead>
<tr>
<th>Problem</th>
<th>Ans</th>
<th>Reason</th>
</tr>
</thead>
</table>
| 26      | .2496 | Event $E = \{ \text{exactly two consecutive heads} \}$

$E = \{ \text{HHTH, HHTT, THTT, TTHH, HTTH} \}$

<table>
<thead>
<tr>
<th>Outcome</th>
<th>HHTH</th>
<th>HHTT</th>
<th>THTT</th>
<th>TTHH</th>
<th>HTTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$wt$</td>
<td>$(.4)^3(.6)$</td>
<td>$(.4)^2(.6)^2$</td>
<td>$(.4)^2(.6)^2$</td>
<td>$(.4)^2(.6)^2$</td>
<td>$(.4)^3(.6)$</td>
</tr>
</tbody>
</table>

$Pr(E) = \frac{\text{sum of } wt}{\text{sum of } wt} = .2496$

27

.305

$E = \{ \text{HHM, HMM, MHH} \}$

$Pr(E) = (.5)(.5)(.3) + (.5)(.5)(.7) + (.5)(.3)(.7) = .305$
No. 3.3
Date 15

Problem Ans Reason

28 (a) \( \frac{47}{147} \)
(b) \( \frac{40}{147} \)

\[ \Pr[\text{2nd ball yellow}] = \frac{2}{7} \cdot \frac{1}{6} + \frac{3}{7} \cdot \frac{2}{6} + \frac{2}{7} \cdot \frac{2}{6} \]
\[ = \frac{2}{7} \left( \frac{1}{6} + \frac{3}{6} + \frac{2}{6} \right) \]
\[ = \frac{2}{7} \left( \frac{6}{6} \right) = \frac{2}{7} \cdot \frac{47}{42} = \frac{47}{147} \]

\[ \Pr[\text{2nd ball red}] = \frac{2}{7} \cdot \frac{5}{6} + \frac{3}{7} \cdot \frac{2}{6} + \frac{2}{7} \cdot \frac{2}{6} \]
\[ = \frac{2}{7} \left( \frac{5}{6} + \frac{2}{6} + \frac{2}{6} \right) \]
\[ = \frac{2}{7} \left( \frac{9}{6} \right) = \frac{2}{7} \cdot \frac{40}{42} = \frac{40}{147} \]

29 \( \frac{27}{98} \)

\[ E = \left\{ 15, 25, 35, 45, 65, 75 \right\} \]

<table>
<thead>
<tr>
<th>outcome</th>
<th>15</th>
<th>25</th>
<th>35</th>
<th>45</th>
<th>65</th>
<th>75</th>
<th>51</th>
<th>52</th>
<th>53</th>
<th>54</th>
<th>56</th>
<th>57</th>
</tr>
</thead>
<tbody>
<tr>
<td>wt</td>
<td>( \frac{1}{7} )</td>
<td>( \frac{1}{7} )</td>
<td>( \frac{1}{7} )</td>
<td>( \frac{1}{7} )</td>
<td>( \frac{1}{7} )</td>
<td>( \frac{1}{7} )</td>
<td>( \frac{1}{7} )</td>
<td>( \frac{1}{7} )</td>
<td>( \frac{1}{7} )</td>
<td>( \frac{1}{7} )</td>
<td>( \frac{1}{7} )</td>
<td>( \frac{1}{7} )</td>
</tr>
</tbody>
</table>

\[ \Pr(E) = \sum \text{wts} = \frac{27}{98} \]

\[ \frac{56}{98} \]
\[
\begin{align*}
\text{Problem 10m} & \quad \text{Ans} \quad \text{Reason} \\
30 & \quad \Pr(E|F) = \frac{\Pr(E \cap F)}{\Pr(F)} \\
& \quad \Pr(E|F') = \frac{\Pr(E \cap F')}{\Pr(F')}
\]

So,

\[
\Pr(E|F) \cdot \Pr(F) + \Pr(E|F') \cdot \Pr(F') = \Pr(E \cap F) + \Pr(E \cap F')
\]

Since \(E \cap F\) and \(E \cap F'\) are disjoint events,

\[
\Pr(E \cap F) + \Pr(E \cap F') = \Pr(E \cap (F \cup F')) = \Pr(E) = 1
\]

Total number of hands: \(\Pr(36,5) = 36 \cdot 35 \cdot 34 \cdot 33 \cdot 32\)

Number of hands with all ranks:

\(36, 32, 28, 24, 20\)

Number of hands with ranks in increasing order:

\[
\begin{bmatrix}
36, 32, 28, 24, 20 \\
36, 35, 34, 33, 32
\end{bmatrix}
\]

\(\text{Ans} = \frac{1}{5!} \)
Sample space $S$ consists of permutations involving 5 cards from deck 100.
Each equally likely:
\[ n(S) = P(100, 5) \]
Event $E$: ranks are consecutive after reordering.
Events in $E$

<table>
<thead>
<tr>
<th>ranks</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4 5</td>
<td>$5! \cdot 10^5$</td>
</tr>
<tr>
<td>2 3 4 5 6</td>
<td>$5! \cdot 10^5$</td>
</tr>
<tr>
<td>3 4 5 6 7</td>
<td>$5! \cdot 10^5$</td>
</tr>
<tr>
<td>4 5 6 7 8</td>
<td>$5! \cdot 10^5$</td>
</tr>
<tr>
<td>5 6 7 8 9</td>
<td>$5! \cdot 10^5$</td>
</tr>
<tr>
<td>6 7 8 9 10</td>
<td>$5! \cdot 10^5$</td>
</tr>
</tbody>
</table>

\[ n(E) = 6! \cdot 10^5 \]

\[ Pr(E) = \frac{6! \cdot 10^5}{100 \cdot 99 \cdot 98 \cdot 97 \cdot 96} \]