

TRIGONOMETRIC INTEGRALS

CHAPTER 7.2

REVIEW

$$\begin{aligned}\frac{d \tan x}{dx} &= \frac{d \sin x}{dx \cos x} \\ &= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x} \\ &= \frac{1}{\cos^2 x} = \boxed{\sec^2 x}\end{aligned}$$

$$\begin{aligned}\frac{d \sec x}{dx} &= \frac{d \frac{1}{\cos x}}{dx} \\ &= (-1) \frac{-\sin x}{\cos^2 x} = \boxed{\sec x \tan x}\end{aligned}$$

$$\tan x = \sin x \sec x$$

$$\begin{aligned}\frac{d \tan x}{dx} &= \frac{d(\sin x \sec x)}{dx} \\ &= \cos x \sec x + \sin x(\sec x \tan x) \\ &= \left(\frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} \right) \sec x \\ &= \boxed{\sec^2 x}\end{aligned}$$

Exercise 7.2.1

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \sin^2(3x) dx &= \int_0^{\frac{\pi}{2}} \frac{1}{2} (1 - \cos(6x)) dx \\ &= \left[\frac{1}{2}x - \frac{1}{12} \sin(6x) \right] \Big|_0^{\frac{\pi}{2}} \\ &= \boxed{\frac{\pi}{4}}\end{aligned}$$

Exercise 7.2.4

$$\int \sin^3 x \, dx = \int \sin x (1 - \cos^2 x) \, dx$$

Substitute $u = \cos x$

$$= - \int (1 - u^2) \, du$$

$$= - \left(u - \frac{1}{3}u^3 \right) + C$$

$$= \boxed{-\cos x + \frac{1}{3}\cos^3 x + C}$$

Exercise 7.2.14

$$I \doteq \int x \sin^3(x^2) dx$$

Substitute $u \doteq x^2$, $du = 2x dx$ and use 7.2.4

$$\begin{aligned} I &= \frac{1}{2} \int \sin^3 u du \\ &= \frac{1}{2} \left(-\cos u + \frac{1}{3} \cos^3 u \right) + C \\ &= \boxed{-\frac{1}{2} \cos(x^2) + \frac{1}{6} \cos^3(x^2) + C} \end{aligned}$$

Exercise 7.2.18

$$\begin{aligned}\int \frac{dx}{1 - \sin x} &= \int \frac{1 + \sin x}{1 - \sin^2 x} dx \\ &= \int \frac{1 + \sin x}{\cos^2 x} dx \\ &= \int \sec^2 dx + \int \frac{\sin x}{\cos^2 x} dx \\ &= \int \sec^2 dx + \int \sec x \tan x dx \\ &= \boxed{\tan x + \sec x + C}\end{aligned}$$

Exercise 7.2.26

$$\begin{aligned}\int \tan^3 x \sec^3 x \, dx &= \\ &= \int \tan x \tan^2 x \sec^3 x \, dx \\ &= \int \tan x (\sec^2 x - 1) \sec^3 x \, dx\end{aligned}$$

Substitute $\sec x \doteq u$

$$\begin{aligned}&= \int (u^2 - 1) u^2 \, du \\ &= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C \\ &= \boxed{\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C}\end{aligned}$$

Exercise 7.2.15

$$\begin{aligned}\int \cos^2 x \tan^3 x \, dx &= \int \sin^2 x \tan x \, dx \\ &= \int (1 - \cos^2 x) \frac{\sin x}{\cos x} \, dx\end{aligned}$$

Substitute $\cos x \doteq u$

$$\begin{aligned}&= \int (1 - u^2) \frac{-du}{u} \\ &= \int \left(-\frac{1}{u} + u\right) du \\ &= -\ln |u| + \frac{1}{2}u^2 + C\end{aligned}$$

$$= \boxed{-\ln |\cos x| + \frac{1}{2} \cos^2 x + C}$$

Exercise 7.2.60 $m > 0, n > 0$ integers

$$\begin{aligned} & \int_{-\pi}^{\pi} \sin(mx) \sin(nx) dx \\ &= \int_{-\pi}^{\pi} \frac{1}{2} (\cos([m-n]x) - \cos([m+n]x)) dx \\ &= \begin{cases} \pi & : m = n \\ \frac{1}{2} \left[\frac{\sin([m-n]x)}{m-n} - \frac{\sin([m+n]x)}{m+n} \right] \Big|_{-\pi}^{\pi} & : m \neq n \end{cases} \\ &= \boxed{\begin{cases} \pi & : m = n \\ 0 & : m \neq n \end{cases}} \end{aligned}$$