1. Consider a probability space with infinitely many coin flips $X_1, X_2, X_3, \ldots$ (i.e. each $X_i$ is 0 or 1.) Consider the sequence of σ-fields $\{F_n\}$ generated by these random variables. Describe the elements of $F_3$.

2. Suppose that $X_i$ are i.i.d random variables with $EX_i = 0, EX_i^2 = 1, EX_i^3 = 0$ and let $S_n = \sum_{i=1}^{n} X_i$. Find a cubic polynomial $f(x)$ (with $n$-dependent coefficients) such that $\{f(S_n)\}$ is a martingale with respect to $\{X_n\}$.

3. Suppose that $\tau_1$ and $\tau_2$ are stopping times with respect to $\{F_n\}$. Show that $\tau_1 + \tau_2$ is also a stopping time.

4. Consider the gambler’s ruin problem with $A = B$, let $\tau$ be the first time we reach $\pm A$. Compute the moment generating function of $\tau$, i.e. $Ee^{\lambda \tau}$.
   Hints: modify the sequence of random variables $e^{tS_n}$ so that you get a martingale and consider the stopped martingale. You will need to use the fact that by symmetry $S_\tau$ is a random variable which is $\pm A$ with probability $1/2-1/2$, and it is independent of $\tau$. 