1. Suppose that \( f : \mathbb{R} \to \mathbb{R} \) is differentiable, prove (without the use of Itô’s formula) that
\[
\int_0^T f(s)dB_s = f(T)B_T - \int_0^T f'(s)B_s ds.
\]

2. (a) Let \( B_t \) be a standard Brownian motion, \( f_1 \) and \( f_2 \) functions on \([0, 1]\) and let \( X_i = \int_0^1 f_i(t)B_t dt \). Find the joint distribution of \( X = (X_1, X_2) \).

(b) Compute the Fourier series of Brownian motion on \([0, 1]\), i.e. compute all finite dimensional distributions of the sequence \( a_n = \int_0^1 \exp(i2\pi nt)B_t dt \).

3. (Exercise 7.1) Find a function \( \tau_t \) such that the processes
\[
X_t = \int_0^t e^s dB_s, \quad Y_t = B_{\tau_t}
\]
have the same distributions. Use this to compute \( EX_t^4 \) and \( P(X_t \geq 1) \).

4. (Exercise 7.2) Show that if \( X_t \) is a continuous local submartingale with
\[
E\left( \sup_{0 \leq s \leq T} |X_s| \right) < \infty
\]
then \( X_t, 0 \leq t \leq T \) is an honest submartingale.
Hint: Try to follow the argument in Proposition 7.10.