1. (a) Suppose that $X(t) = (X_1(t), X_2(t)) \in \mathbb{R}^2$ satisfies
\[
\begin{align*}
    dX_1 &= -X_2 dB \\
    dX_2 &= X_1 dB
\end{align*}
\]
and $|X(0)| = 1$. Show that $X(t)$ does not stay on the circle $|X| = 1$, even though the vector field $(-X_2, X_1)$ is tangent to the circle at $(X_1, X_2)$.
(Hint: Apply Itô to $|X(t)|^2$.)
(b) Show that if $X(t) = (X_1(t), X_2(t))$ satisfies
\[
\begin{align*}
    dX_1 &= -X_2 dB - \frac{1}{2} X_1 dt \\
    dX_2 &= X_1 dB - \frac{1}{2} X_2 dt
\end{align*}
\]
with $|X(0)| = 1$ then $X(t)$ stays on the unit circle.

2. Show that if $B_t$ is a standard BM then $M_t = (B_t + t) \exp(-B_t - t/2)$ is a martingale.

3. **(Exercise 9.2)** Solve the SDE
   \[
   dX_t = tX_t dt + e^{t^2/2} dB_t, \quad X_0 = 1. 
   \]

4. **(Exercise 9.3)** Solve the SDE
   \[
   X_0 = 0, \quad dX_t = -\frac{2X_t}{1-t} dt + \sqrt{t(1-t)} dB_t, \quad 0 \leq t \leq 1. 
   \]
   Show that the solution is a Gaussian process and find the covariance function $\text{Cov}(X_s, X_t)$.

**Bonus problem.**

**(Exercise 8.5)** Let $B(t)$ be a 3-dimensional standard BM. Let $f(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$, show that $M_t = f(B_t), 1 \leq t < \infty$ is an $L^2$ bounded local martingale, but not a martingale.