Homework 2

Due: March 27, 2014.

1. Solve Exercise 3.8 from the book.

2. Let $B(t)$ be a standard Brownian motion. For $x \in \mathbb{R}$ consider the process

$$Y_t^{(x)} = xe^{-t} + B(1 - e^{-2t}).$$

Show that this is a Feller process and prove that its generator is given by

$$\mathcal{L}f = f'' - xf'(x), \quad \mathcal{D}(\mathcal{L}) = \{f \in C(\mathbb{R}) : f', f'' \in C(\mathbb{R})\}.$$

(Here $C(\mathbb{R})$ means the set of continuous functions vanishing at infinity.)

3. Give an example for a linear operator $\mathcal{L}$ on $C[0,1]$ which is not closable.

4. Solve Exercise 3.21 from the book.