1. [9.1 #26]

(1) \( x = \sin 3u, \ y = \sin 4u \) is such that \(|x|, |y| \leq 1 \) so the plot is bounded and has to be either I, II or V. At \( u = 0, \ x = y = 0 \) so it has to be V.

(2) \( x = u^4 - u^2, \ y = u + \ln u \) is such that \( y \to -\infty \) as \( u \to 0^+ \) in which case \( x \to 0 \). The only curve for which this is true is VI.

2. [9.1 # 36] Find parametric equations for the set of all points \( P \) determined as shown in the figure so that \(|OP| = |AB|\). Sketch the curve.

Let \( \theta \) be the angle between the \( x \)-axis and the radial line \( OB \), then \(|OB| = 2a/\cos \theta \) and \(|OA| = 2a \cos \theta \) so \(|OP| = |OB| - |OA|\). So in polar coordinates:

\[
|OP| = r = 2a \left( \frac{1}{\cos \theta} - \cos \theta \right) = 2a \frac{\sin^2 \theta}{\cos \theta},
\]

or in Cartesian coordinates

\[
x = 2a \sin^2 \theta, \quad y = 2a \frac{\sin^3 \theta}{\cos \theta}.
\]

As \( \theta \to \pi/2 \) we see that \( y \to \infty \) and \( x \to 2a \), so the curve asymptotes to the vertical \( x = 2a \).

For \( \theta \ll 1 \), the first terms of the Taylor series expansion of \( \sin \theta \) and \( \cos \theta \) give \( x \approx 2a \theta^2 \), \( y \approx 2a \theta^3 \) so \( \frac{y}{2a} \approx \left( \frac{x}{2a} \right)^{3/2} \), near \( x, y = 0 \).

3. [9.5 # 9] \( r^2 = 4 \cos 2\theta \)

Need \( \cos 2\theta \geq 0 \), so \( \theta \) is restricted to the ranges \(-\pi/4 \leq \theta \leq \pi/4 \) and \(3\pi/4 \leq \theta \leq 5\pi/4 \) (up to a factor of \( 2\pi \)). These two ranges in fact give portions of the curve that are mirror images across the \( y \)-axis. Furthermore, \( r \leq 2 \), so the curve lives inside the disk of radius 2 and in the angular sectors \(-\pi/4 \leq \theta \leq \pi/4 \) and \(3\pi/4 \leq \theta \leq 5\pi/4 \). Now \( r(\pi/4) = r(-\pi/4) = 0 \) so the curve intersects itself at the origin. The curve looks like a figure 8 lying on its side. The area it encloses is

\[
A = 2 \int_{-\pi/4}^{\pi/4} \frac{r^2}{2} d\theta = 4 \int_{-\pi/4}^{\pi/4} \cos 2\theta d\theta = 4.
\]

In Cartesian coordinates: \( r^2 = 4 \cos 2\theta = 4(\cos^2 \theta - \sin^2 \theta) \) so \( r^4 = 4r^2 \cos^2 \theta - 4r^2 \sin^2 \theta \), i.e.

\[
(x^2 + y^2)^2 = 4x^2 - 4y^2.
\]

or \( x^2 = y^2 + (x^2 + y^2)^2/4 \) and near \( x = y = 0 \), where the curve intersect itself, \( x^2 \approx y^2 \) or \( x \approx \pm y \), so the two tangents at the intersection have slopes \( \pm 1 \). Another way to get the slopes: \( x = r \cos \theta, \ y = r \sin \theta \), so near \( \theta = \pi/4 \) we have \( \cos \theta \approx \cos \pi/4 = 1/\sqrt{2} \) and \( \sin \theta \approx \sin \pi/4 = 1/\sqrt{2} \) and \( x \approx y \). Similarly, near \( \theta = -\pi/4, \ x \approx -y \).
4. (a) \( \frac{1}{1 + 1}, \frac{1}{1 + \frac{1}{1 + 1}}, \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + 1}}}, \ldots \)

this is the sequence \( a_1 = 1, a_{n+1} = \frac{1}{1 + a_n} \). If it converges the limit must satisfy \( L = \frac{1}{1 + L} \), so \( L = (\sqrt{5} - 1)/2 \).

(b) \( \lim_{n \to \infty} \frac{1 + 2^3 + 3^3 + 4^3 + \cdots + n^3}{n^4} = 1/4 \)

(c) \( \sum_{n=0}^{\infty} (-2)^n \), diverges. \( a_n \) does NOT \( \to 0 \) as \( n \to \infty \). This is a geometric series with \( q = (-2) \) and \( |q| > 1 \).

(d) \( \sum_{n=0}^{\infty} \frac{3^n}{n!} = e^3 \).

5. Starting with a square with sides of length \( L \) join the middle of each edge to create a new square, then join the middle of the edges of that new square to create another and so on indefinitely. What is the sum of the areas of all the squares? (no \( \sum \) in your answer).

The first square has area \( L^2 \). The next square has half the area, the third has half the area of the second etc... so the total area is

\[
A = L^2 \left( 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots \right) = L^2 \frac{1}{1 - \frac{1}{2}} = 2L^2.
\]