Math 221: Calculus and Analytic Geometry

TEXTBOOK: *Calculus and Analytic Geometry*, Thomas & Finney, 5th Ed.

Math 221 covers the first 6 chapters of Thomas and Finney, 5th. Roughly speaking, 1000 students take Math 221, both in the Fall and in the Spring semester. Most of these students will become users of mathematics, not mathematicians, and are taking the 221-222-234 sequence because it is required for their degree and subsequent classes in other departments. It is therefore important that lecturers follow the syllabus closely. **Substantial departure from the syllabus requires advance approval from the Calculus committee.**

Notes and recommendations for Instructors

When covering section 3-10, emphasize that the “extended mean value theorem” is known as “Taylor’s formula” and use Taylor’s formula as often as possible (as an alternative to l’Hôpital’s Rule for instance). Taylor’s formula is very important for applications and as preparation for treatment of series in 222. Taylor’s formula leads to deeper understanding of the concepts while l’Hôpital’s rule leads to thoughtless applications of formulas. L’Hôpital’s rule is very useful for limits involving logs and exponentials however.

When covering integrals, discuss the various incarnations of the fundamental theorem and the notation subtleties including the concept of “dummy variables”:

\[ \int f(x) \, dx, \int_a^x f(s) \, ds, \int \frac{dF}{dx} \, dx, \int dF, \int f'(x) \, dx, \int_a^x f'(s) \, ds, \frac{d}{dx} \int_a^x f(s) \, ds, \ldots \]

An example from linear differential equations which confuses most students is \( \frac{d}{dx} \int_a^x f(x) g(s) \, ds \). When using substitution, students should know how to change the limits of integration on definite integrals, this is important for applications and is often easier computationally. Note that the book does not mention the method of substitution explicitly. It is good to correct that.

Section 4-6 is conceptually useful to help explain dummy variables and as an introduction to the fundamental theorem if telescoping sums are employed (e.g. \( \sum_{j=1}^n j = \sum_{j=1}^n ((j+1)^2 - j^2)/2 \)), but it may also be confusing to students who might think that they must compute elementary integrals as limits of sums. In practice, the connection between sums and integrals is used in the opposite manner: using integrals to estimate sums. Make sure you leave enough time to cover logs and exponentials properly at the end of the semester. Note that the book uses the \( e^x \) notation right away. It is good to use also the notation \( \exp(x) \) and it is nice to deduce that \( \exp(x) \equiv e^x \).
Math 221 Syllabus: Calculus and Analytic Geometry
(assumes 15 weeks of instruction)

Week 1
1-1,2,3,4,5 Coordinates and lines, 1-6 Functions and graphs,
1-7,8,9 Tangent and Velocity.

Week 2
1-10, 2-11 Limits and continuity, 2-1,2 The derivative

Week 3
2-3 Differentiation formulas,
2-4,5 Inverse and Implicit functions, (6-1,2,3 Inverse trig functions?)

Week 4
2-8 Composite functions, chain rule, 2-9,10; 6-2,3 Differentiating trig functions.

Week 5
2-6, 2-12,13 Increments, differentials and higher derivatives

Week 6
3-2 Related Rates,
3-1,3,4 Curve plotting

Week 7
3-5,6 Max Min,
3-7,8 Mean Value Theorem

Week 8
3-10 Taylor’s formula,
3-9 Indeterminate forms (l’Hôpital’s Rule)

Week 9
4-2,3,4 Indefinite integrals (antiderivatives) and differential equations

Week 10
4-5,(6),7 The definite integral (area under a curve),
4-8 The Fundamental Theorem

Week 11
4-9,10 Trapezoidal rule, notation and summary, 5-1,2 Plane Areas,
5-3 Distance, 5-9 Average value.

Week 12
5-4,5 Volumes

Week 13
6-1,2,3 Inverse Trig Functions
6-4,5,6,7 The natural log

Week 14
6-8,9,10 Log and exponential functions
6-11 Exponential growth (the equation \( y’ = y \)).

Week 15
Review and/or further applications of integration: (5-7,8,10,11 Arclength, Surfaces, Center of mass)