Mathematics 211, Lecture 2 (Wilson) Your Name: ________________

Circle your TA’s name:

Mustafa Erdem Michael Lang Simon MacNair
Wenjun Qiu Yoomi Rho Julia Velikina

Exam II 4/2/96

Write your answers to the seven problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\pi$, $e$, $\sqrt{3}$, $\ln(2)$, and similar numbers) rather than using decimal approximations.

You may refer to notes you have brought in on one sheet of paper, as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

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Problem 1  
(10 points)

Suppose that you can predict inflation and know that the price of a movie ticket fifteen years from now will be $88. Suppose also that you know that you could invest money, compounded continuously, at 13% for those fifteen years. What is the present value of one of those movie tickets?
Problem 2 \hspace{0.5cm} (13 points)

Assume $y$ is a differentiable function of $x$ such that $x$ and $y$ satisfy the relation

$$\sqrt{x^2 + y^2} + 19 = 2xy$$

Find an equation for the tangent line to the graph of $y(x)$ at the point $(3, 4)$. 
Problem 3  (16 points)

Find the derivatives $\frac{dy}{dx}$ for the given functions $y(x)$:

(a) $y = e^{(x^2 - 2x)}$

(b) $y = \ln(x^2 + 2x - 5)$

(c) $y = (3x + 1)^{2x}$

(d) $y = \log_4(x^3)$
Problem 4  (16 points)

Evaluate the indefinite integrals:

(a) 
\[ \int (x + 3)(5x - 9)dx \]

(b) 
\[ \int \sqrt{x^4 - 2x^2} (x^3 - x)dx \]

(c) 
\[ \int \frac{3x}{\sqrt{x^2 + 5}} dx \]

(d) 
\[ \int e^{-3x}dx \]
Problem 5  (15 points)

The number of unicorns in a population at time $t$ (in years) is given by $P(t)$. The rate of growth of the population is observed to be proportional to the population. The number of unicorns 3 years after observations are started is 2000, while 9 years after observations are started the number has declined to 500.

(a) Find a function $P(t)$ giving the population for every time $t$.

(b) What will be the population when observations have been under way 12 years?

(c) When is the population 1000?
Problem 6  (15 points)

A balloon is released from the ground and rises vertically at a constant rate of 8 feet per second. An observer is 10 feet from the balloon at the instant it starts to rise, and walks straight toward the balloon at a rate of 4 feet per second. How fast is the distance between the balloon and the observer changing one second after the balloon is released?
Problem 7  (15 points)

$y(x)$ is a function such that $y'(x) = 3x^2 + 3$ and $y(2) = 18$. Find $y(x)$. 