Circle your TA’s name:

- Joni Baker
- John Brown
- Brian Curtin
- Stephanie Edwards
- Cheryl Good
- Jiansheng Huang
- Vladimir Yegorov

Exam I 2/28/95

- Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure
  - to tell where to look for the answer, and
  - to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

- You may use your calculator in doing these problems. If you do some of the work using a calculator, however, be sure to tell precisely what you asked the calculator to do. On all answers be sure to show your work.

- Unsupported answers, even if they give the correct final answer, may receive little or no credit.

- Wherever possible, (even in calculator-assisted answers!) leave your answers in exact forms (using $\pi$, $e$, $\sqrt{3}$, $\ln(2)$, and similar numbers) rather than using decimal approximations.

- There is scratch paper attached. If you need more scratch paper, please ask for it.

- You may refer to notes you have brought in on one sheet of paper.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Problem 1 (12 points)
Evaluate the following integrals:

(a) \[ \int_{0}^{1} \frac{x^2}{x^2 + 1} \, dx \]

(b) \[ \int_{0}^{\pi} \frac{\tan(x) - \sec(x)}{\cos(x)} \, dx \]
Problem 2  (14 points)
Evaluate the following integrals:
(a) \[
\int_{0}^{\infty} \frac{1}{(x-1)^2} \, dx
\]

(b) \[
\int e^x \sin(2x) \, dx
\]
Problem 3  (16 points)
Evaluate the following integrals:
(a) \[ \int e^{\sqrt{x}} \, dx \]
(b) \[ \int_{0}^{\pi} \sin^2 x \cos^3 x \, dx \]
Problem 4  (12 points)
Evaluate the integral:
\[ \int \frac{x^2}{\sqrt{1 - x^2}} dx \]
Problem 5   (12 points)
Evaluate the integral:
\[ \int \frac{6x^2 - 16x + 14}{(x^2 - 2x + 2)(x - 3)} \, dx \]
Problem 6  (10 points)
For each of the sequences $a_n$ below, tell whether it converges and,
(i) if it converges, tell the limit:
(ii) if it does NOT converge, tell how you know that.
You do NOT need to give a formal proof of convergence or divergence using epsilon and N.

(a) 
\[ a_n = \frac{n + 1}{1 + \sqrt{n}} \]

(b) 
\[ a_n = \left(1 + \frac{2}{n}\right)^n \]
Problem 7  (12 points)
For each of these series, tell whether it converges and give a reason for your answer. IF IT DOES CONVERGE, what is the sum?

(a) \[ \sum_{n=1}^{\infty} \cos(n\pi) \]

(b) \[ \sum_{n=1}^{\infty} \frac{2^{n+1}}{7^n} \]
Problem 8  (12 points)
For each of these series, tell whether it converges and give a reason for your answer. You do NOT have to tell WHAT it converges to, if it does converge.

(a) \[ \sum_{n=1}^{\infty} \frac{1}{1 + n\sqrt{n}} \]

(b) \[ \sum_{n=1}^{\infty} \frac{2^n}{n!} \]