Mathematics 222, Lecture 2 (Wilson)  Your Name: ________________________
Circle your TA’s name:

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Cheryl Grood  Jiansheng Huang  Vladimir Yegorov

Final Examination  Spring 1995

- THERE ARE ELEVEN PROBLEMS, INCLUDING ONE ON THE BACK OF THIS SHEET!

- Write your answers to the eleven problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure to tell where to look for the answer, and to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

- Unsupported answers, even if they give the correct final answer, may receive little or no credit. You may use your calculator in doing these problems. If you do some of the work using a calculator, however, be sure to tell precisely what you asked the calculator to do.

- Wherever possible (even in calculator-assisted answers!) leave your answers in exact forms (using $\pi$, $e$, $\sqrt{3}$, $\ln(2)$, and similar numbers) rather than using decimal approximations.

- You may refer to notes you have brought in on up to three sheets of paper.

- If you have to write down a series, be sure to tell what its terms look like in general: Don’t just give a few examples.

- Be sure to use some notation that makes clear which things you write are vectors and which are numbers.

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Problem 1  (20 points)
Evaluate the integrals:
(a) \[ \int x^2 \cos(x) \, dx \]
(b) \[ \int \frac{14x^2 - 14x + 5}{(2x^2 - 2x + 1)(2x - 1)} \, dx \]
Problem 2  (20 points)
Evaluate the integrals:
(a) \[ \int_{-\pi}^{0} \tan^3 x \, dx \]
(b) \[ \int_{0}^{\infty} \frac{1}{1 + x^2} \, dx \]
Problem 3  (16 points)

(a) Does the series
\[
\sum_{n=1}^{\infty} \frac{1 + \sin n}{n^2 - 50}
\]
converge? Be sure to give reasons!

(b) The series
\[
\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \ldots
\]
converges for \(-\frac{\pi}{2} < x < \frac{\pi}{2}\). Use this fact to find the first five terms of a series for \(\sec^2 x\).
Problem 4 (20 points)

Let $f(x) = \cos(x^2)$.

(a) Write the Maclaurin series for $f(x)$. Show where the coefficients come from: Do not simply “plug” $x^2$ into the series for $\cos(x)$.

(b) Use the series you got in (a) to estimate

$$\int_{0}^{1} f(x) \, dx$$

with an error of magnitude at most $10^{-4}$. 

Problem 5  (16 points)

(a) Find the tangent line at the point where \( t = \frac{\pi}{4} \) on the curve \( x = 1 + \tan^2 t, \ y = 2 + \sec^2 t \).

(b) Find the value of \( \frac{d^2y}{dx^2} \) at the point specified above.
Problem 6  (16 points)
Find the area of the region which is inside $r = 8$ and to the right of $r = 4\sec \theta$. 
Problem 7  (16 points)
Let $\vec{A} = 3\vec{i} - 2\vec{j} + \vec{k}$ and $\vec{B} = 3\vec{i} - 7\vec{j} + 5\vec{k}$. Express $\vec{B}$ as the sum of two vectors, one in the direction of $\vec{A}$ and one perpendicular to $\vec{A}$. Be sure to identify which is which!
Problem 8  (18 points)
Consider the three points in space $P = (1, -1, 2)$, $Q = (3, 0, -1)$, and $R = (2, 1, 3)$.
(a)  Find the area of a parallelogram three of whose vertices are $P$, $Q$, and $R$.

(b)  Find a vector of unit length which is perpendicular to the plane containing $P$, $Q$, and $R$. 
Problem 9  (18 points)
An object moves with position vector at time \( t \) given by \( \vec{r}(t) = (t - \sin t)\hat{i} + (1 - \cos t)\hat{j} \).
(a) Find the velocity vector for this motion.

(b) Find the acceleration vector for this motion.

(c) Find the angle between the velocity and acceleration at the time \( t = \frac{\pi}{6} \).
Problem 10  (20 points)
Movement along a curve is parametrized by the position vector \( \mathbf{r}(t) = (3 \cos t)\mathbf{i} + (3 \sin t)\mathbf{j} - (2t^3)\mathbf{k} \).

(a) Find distance traveled as a function of time, \( s(t) \).

(b) Find the length of this curve from \( t = 0 \) to \( t = 3 \).
Problem 11  \hspace{0.5cm} (20 points)

Let position be given by the vector \( \vec{r}(t) = (e^t \sin 2t)\hat{i} + (e^t \cos 2t)\hat{j} + 2e^t\hat{k} \).

(a) Find the curvature \( \kappa \) at the instant when \( t = 0 \).

(b) Find the unit tangent vector \( \vec{T} \) at the instant when \( t = 0 \).

(c) Find the principal unit normal vector \( \vec{N} \) at the instant when \( t = 0 \).

(d) Find the unit binormal vector \( \vec{B} \) at the instant when \( t = 0 \).