THERE IS A PROBLEM ON THE BACK OF THIS SHEET!

Write your answers to the six problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\pi$, $\sqrt{3}$, $\cos^{-1}(2)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is scratch paper at the end of the exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on a sheet of paper, as announced in class.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. (“I did it on my calculator” is not sufficient substantiation...)

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Problem 1  (14 points)
Find an equation for the ellipse that:

(a) has one focus at \((0, 0)\),

(b) crosses the x-axis at \((-2, 0)\) (and perhaps at other points as well), and

(c) has its other focus on the positive x-axis.

Problem 2  (15 points)

(a) Sketch the graph of \(r = 2\cos(3\theta)\). (Be sure to mark on the axes what your unit length is.)

(b) Find the angle \(\psi\) between the radius vector and the tangent line to the graph of \(r = 2\cos(3\theta)\) at the point where \(\theta = \frac{\pi}{7}\).
Problem 3 (24 points)
Evaluate:

(a) \[ \int \frac{x - 5}{(x + 2)(x - 1)} \, dx \]

(b) \[ \int_{-1}^{3} \frac{1}{\sqrt{3} - x} \, dx \]

(c) \[ \int \frac{x^2}{\sqrt{1 - x^2}} \, dx \]

(d) \[ \int e^{2x} \cos(2x) \, dx \]
Problem 4  (16 points)
Consider the graph of
\[ x^2 + 4xy + 5y^2 + 2x - 10y = 11 \]
(a) What kind of curve is this? (ellipse, parabola, hyperbola)

(b) Through what angle \( \alpha \) should the coordinates be rotated so as to eliminate the \( xy \) term in this equation?

(c) Find equations for the tangent and normal lines to this graph at the point \((1, 2)\).

Problem 5  (16 points)
Find the area of the region which is inside both the circle \( r = \cos(\theta) \) and the circle \( r = -\sin(\theta) \). (I.e., the region where the two circular disks overlap.)
Problem 6  (15 points)
For the hyperbola \( \frac{x^2}{13^2} - \frac{y^2}{5^2} = 1 \):

(a) Give the coordinates of both foci.

(b) Give the coordinates of any points where the curve crosses the coordinate axes.

(c) Find equations for the two asymptotes.

(d) Sketch the graph. Be sure to indicate what you use as unit length.