Your Name: ________________________________

Circle your TA’s name:
Mark MacLean   Adnan Rebei   Peter Wiles

Exam II       11/19/98

Write your answers to the six problems in the spaces provided. If you must continue
an answer somewhere other than immediately after the problem statement, be sure
(a) to tell where to look for the answer, and (b) to label the answer wherever it winds
up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using π, √3, and similar
numbers) rather than using decimal approximations.

Some formulas which might be useful: \( \sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta)) \). \( \cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta)) \).

\[ \int u \, dv = uv - \int v \, du. \]

You may refer to notes you have brought in on one sheet of paper (regular notebook
or typing size) as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO
CREDIT FOR UNSUBSTANTIATED ANSWERS.

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Problem 1  (17 points)
The equation $\rho = 2\sin(\phi)$ in spherical coordinates describes a “donut” as shown. Find its volume.
Problem 2  (16 points)
Set up the following integral with the order of integration reversed, and then evaluate the resulting integral.

\[
\int_{0}^{2} \int_{0}^{4-x^2} 2x \ dy \ dx
\]
Problem 3   (17 points)
A thin plate is bounded by the $y$-axis, the line $y = x$, and the line $y = 2 - x$. It is made of material with density $\delta(x, y) = 6x + 3y + 3$. Find its mass and its center of mass.
Problem 4 (17 points)
Let $E$ be the wedge cut from the cylinder $x^2 + y^2 \leq 1$ by the planes $z = 0$ and $z = -y$. Evaluate
\[
\iiint_E 2x \, dv.
\]
Problem 5  (16 points)
Evaluate the line integral of \( f(x, y, z) = x + y + z \) along the straight line segment from \((1, 2, 3)\) to \((0, -1, -1)\).
Problem 6  (17 points)
Let $C$ be the path traversed by $\vec{r}(t) = t \vec{i} + t^2 \vec{j} + t^4 \vec{k}$ for $0 \leq t \leq 1$.
Let $\vec{F}(x, y, z) = \sqrt{z} \vec{i} - 2x \vec{j} + \sqrt{y} \vec{k}$.
Evaluate the line integral of $\vec{F}$ along $C$,

$$\int_C \vec{F} \cdot d\vec{r}.$$