

Double Integral

1. True or False: Let $D = \{(x, y) : a \leq x \leq b, c \leq y \leq d\}$

(a) $\iint_D f(x, y) + g(x, y) dA = \iint_D f(x, y) dA + \iint_D g(x, y) dA$

(b) $\iint_D f(x, y)g(x, y) dA = \iint_D f(x, y) dA \iint_D g(x, y) dA$

(c) $\iint_D f(x)g(y) dA = \int_a^b f(x) dx \int_c^d g(y) dy$

2.

$$\int_0^1 \int_{-y}^y x^2 dx dy$$

3.

$$\int_0^1 \int_y^1 x^2 e^{xy} dx dy$$

4. Calculate

$$\iint_D \frac{\sin x}{x} dA,$$

where D is the triangle in the xy -plane bounded by the x -axis, the line $y = x$, and the line $x = 1$.

5. Consider the functions $f(x) = \sin x$ and $g(x) = \cos x$

(a) Find the two intersection points when $x \in [0, 2\pi]$

(b) Compute the area of the region between the two curves when $x \in [\frac{\pi}{4}, \frac{5\pi}{4}]$.

6. Consider the two level sets $f(x, y) = x - y^2 = 0$ and $g(x, y) = x^2 + y^2 = 2$

(a) Find the two intersection points.

(b) Compute the area of the region bounded by these two points and the curves between them.

Solutions:

1. (a) True

(b) False

(c) True

2. $1/6$

3. $\frac{e}{2} - 1$

4. $1 - \cos 1$

5. (a) $(\frac{\pi}{4}, \frac{\sqrt{2}}{2}), (\frac{5\pi}{4}, -\frac{\sqrt{2}}{2})$

(b) $2\sqrt{2}$

6. $\sqrt{2} + \frac{\pi}{2} - \frac{2}{3}$