

Function of Two Variables

1. $x + 2y - 12 = 0$

(a) What does this equation represent in the 3-dimensional space? What in the 2-dimensional space?

(b) Now consider this equation in the 3-dimensional space. Find the intersections of this equation with the coordinate axes.

(c) Still in the 3-dimensional space. Find the intersections of the equation with (x, y) -plane, (y, z) -plane and (x, z) -plane.

2. Classify the following quadratic forms:

(a) $f_1(x, y) = x^2 + 2xy$

(b) $f_2(x, y) = 4x^2 + 7xy + 5y^2$

(c) $f_3(x, y) = x^2 + 6xy + 9y^2$

3. (a) Show that if a quadratic form $f(x, y) = Ax^2 + Bxy + Cy^2$ can be written as $(ax + by)(cx + dy)$ with $ad \neq bc$, then $f(x, y)$ is indefinite.

(b) In the first part, if $ad = bc$, is $f(x, y)$ still indefinite?

4. For the following equations in Cartesian coordinates (x, y) , find their equation in Polar Coordinates (r, θ) .

(a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(b) $y = 4x^2$

5. Describe the graph for each of the following functions

(a) $z = r^2$

(b) $z = \cos \theta$

(c) $z^2 = 1 - r^2$

Solutions:

1. (a) A plane. A line.

(b) x-axis: $(2, 0, 0)$. y-axis: $(0, 6, 0)$. z-axis: no intersection.

(c) x,y-plane: $\begin{pmatrix} 12 - 2t \\ t \\ 0 \end{pmatrix}$.

y,z-plane: $\begin{pmatrix} 0 \\ 6 \\ t \end{pmatrix}$.

x,z-plane: $\begin{pmatrix} 12 \\ 0 \\ t \end{pmatrix}$.

2. (a) Indefinite

(b) Definite

(c) Semidefinite

3.

4. (a) $\frac{r^2 \cos^2 \theta}{a^2} + \frac{r^2 \sin^2 \theta}{b^2} = 1$

(b) $r \sin \theta = 4r^2 \cos^2 \theta$

5.