Line Integrals of Vector Fields

1. Evaluate $\int_C \sqrt{x+y}dx$, where C is the union of three line segments from $(0,0)$ to $(1,3)$, from $(1,3)$ to $(0,3)$, and from $(0,3)$ to $(0,0)$.

2. Along the curve $\vec{x}(t) = t\vec{i} - \vec{j} + t^2\vec{k}$, $0 \leq t \leq 1$, evaluate each of the following integrals.

   (a) $\int_C (x+y-z)dx$

   (b) $\int_C (x+y-z)dy$

   (c) $\int_C (x+y-z)dz$

3. Along the curve $\vec{x}(t) = (\cos t)\vec{i} + (\sin t)\vec{j} - (\cos t)\vec{k}$, $0 \leq t \leq \pi$, evaluate each of the following integrals:

   (a) $\int_C xzdx$

   (b) $\int_C xzdy$

   (c) $\int_C xyzdz$

4. Evaluate $\int_C \vec{f}d\vec{x}$, where $\vec{f}(x, y, z) = z\vec{i} + xy\vec{j} - y^2\vec{k}$ along the curve C given by $\vec{x}(t) = t^2\vec{i} + t\vec{j} + \sqrt{t}\vec{k}, 0 \leq t \leq 1$.

5. Evaluate the line integral $\int_C -ydx + zdy + 2xdz$, where C is the helix $\vec{x}(t) = (\cos t)\vec{i} + (\sin t)\vec{j} + t\vec{k}, 0 \leq t \leq 2\pi$.

Solutions:

1. $-4 + 2\sqrt{3}$
2. (a) $-\frac{5}{6}$
   (b) 0
   (c) $-\frac{5}{6}$
3. (a) $\frac{2}{3}$
   (b) 0
   (c) 0
4. $\frac{17}{20}$
5. π