MATH 234 QUIZ 06 OCTOBER 2014

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Problem 1 (7 points):

1. Draw the zero set of the function $f(x, y) = \sin(x)\sin(y)$
2. Find all critical points of $f(x, y)$. Which are local minima or local maxima.

Solution:

1. $f(x, y) = 0 \iff \sin x = 0$ or $\sin y = 0$. Notice that $\sin x = 0$ if and only if $x = k\pi$ for some integer k. Thus the zero set of f are all the vertical lines $x = k\pi$ and all the horizontal lines $y = k\pi$ where k is any integer.

2. $$f_x = \cos(x)\sin(y) = 0 \implies x = (k + \frac{1}{2})\pi \text{ or } y = k\pi$$
$$f_y = \sin(x)\cos(y) = 0 \implies x = k\pi \text{ or } y = (k + \frac{1}{2})\pi$$

   So critical points of f are $(x, y) = (m\pi, n\pi)$ or $((m+\frac{1}{2})\pi, (n+\frac{1}{2})\pi)$. It is easy to see that $f(x, y)$ has maximal value 1 and minimal value -1. And $f(x, y) = 1$ if and only if $\sin x = \sin y = 1$ or $\sin x = \sin y = -1$. So the maximum points are $((2m + \frac{1}{2})\pi, (2n + \frac{1}{2})\pi)$ and $((2m+\frac{3}{2})\pi, (2n+\frac{3}{2})\pi)$. Similarly, $f(x, y) = -1$ if and only if $(x, y) = ((2m+\frac{1}{2})\pi, (2n+\frac{3}{2})\pi)$ or $((2m + \frac{3}{2})\pi, (2n + \frac{1}{2})\pi)$, which gives all the minimum points. Here, m and n represent any integers.

Problem 2 (3 points): Find the critical point of $f(x, y) = x^2 + 9y^2 + 2x - 6y - 10$. Is it a local minimum or local maximum?
Solution: To find critical points, solve the equations $\nabla f = 0$.

$$f_x = 2x + 2 = 0 \implies x = -1$$
$$f_y = 18y - 6 = 0 \implies y = \frac{1}{3}$$

So there is only one critical point $(-1, \frac{1}{3})$. To classify this critical point, notice that

$$\begin{aligned} f(x, y) &= (x^2 + 2x + 1) + (9y^2 - 6y + 1) - 12 \\ &= (x + 1)^2 + (3y - 1)^2 - 12 \\ &\geq -12 \end{aligned}$$

Since at the critical point $f(-1, \frac{1}{3}) = -12$, it is the minimum.