MATH 234 QUIZ 07 OCTOBER 2014

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Problem 1 (4 points): The second order Taylor expansion of a function $z = f(x, y)$ at a point $(2, 1)$ is given by

$$f(x, y) = 3 + 2\Delta x + 3\Delta y + 12(\Delta x)^2 - \Delta x\Delta y + (\Delta y)^2 + \cdots$$

where $\Delta x = x - 2$ and $\Delta y = y - 1$.
Which of the following statements is true?

1. the function has a local minimum at $(2, 1)$.
2. the function has a saddle point at $(2, 1)$.
3. $(2, 1)$ is not a critical point of the function.

Explain your choice.
Solution:
The third one is correct, since the first partial derivatives are not zero.

Problem 2 (6 points): Find all critical points of the following functions, and apply the second derivative test to the points you find.

1. $f(x, y) = x^2 + y^2 - \frac{1}{2}xy^2$
2. $f(x, y) = x + 2y - xy^2$

Solution:

1. $f_x = 2x - \frac{1}{2}y^2 = 0$ and $f_y = 2y - xy = 0$ give three critical points $(0, 0)$, $(2, 2\sqrt{2})$ and $(2, -2\sqrt{2})$. Then apply the second derivative test to each point.

$$\begin{aligned} f_{xx} &= 2 \\ f_{xy} &= -y \\ f_{yy} &= 2 - x \end{aligned}$$

At $(0, 0)$, $D = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 4 > 0$ and $f_{xx}(0, 0) = 2 > 0$. Thus $(0, 0)$ is a local minimum.

At $(2, \pm 2\sqrt{2})$, $D = \begin{pmatrix} 2 & \mp 2\sqrt{2} \\ \mp 2\sqrt{2} & 2 \end{pmatrix} = -4 < 0$. Thus $(2, \pm 2\sqrt{2})$ are saddle points.

2. $f_x = 1 - y^2 = 0$ and $f_y = 2 - 2xy = 0$ give two critical points $(1, 1)$ and $(-1, -1)$. Then apply the second derivative test to each point.

$$\begin{aligned} f_{xx} &= 0 \\ f_{xy} &= -2y \\ f_{yy} &= -2x \end{aligned}$$

At $(1, 1)$, $D = \begin{pmatrix} 0 & -2 \\ -2 & -2 \end{pmatrix} = -4 < 0$. Thus $(1, 1)$ is a saddle point.

At $(-1. -1)$, $D = \begin{pmatrix} 0 & 2 \\ 2 & 2 \end{pmatrix} = -4 < 0$. Thus $(-1, -1)$ is a saddle point.