MATH 234 QUIZ 08 OCTOBER 2014

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Problem 1 (5 points): Find all points on the surface

$$xy - z^2 + 1 = 0$$

that are closest to the origin.

Solution: Let $f(x) = x^2 + y^2 + z^2$ be the square of the distance, and constraint set $g(x, y, z) = xy - z^2 + 1 = 0$. Then

$$\nabla f = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}, \nabla g = \begin{pmatrix} y \\ x \\ -2z \end{pmatrix}.$$

Use Lagrange Multiplier Method to set up equations:

$$\begin{cases} 2x = \lambda y \\ 2y = \lambda x \\ 2z = -2\lambda z \\ xy - z^2 + 1 = 0 \end{cases}$$

The third equation gives $z = 0$ or $\lambda = -1$.

If $z = 0$, then $xy = -1$. So $2x = \lambda y = \lambda(\lambda y/2)$ and $y \neq 0$ implies $\lambda = \pm 2$. If $\lambda = 2$, then $x = y$, which is not possible because $xy = -1$. If $\lambda = -2$, then you get two possible minimum points $(1, -1, 0)$ and $(-1, 1, 0)$.

If $\lambda = -1$, then $2x = -2y = -2(-2x)$ implies $y = 0$ and hence $x = 0, z = \pm 1$. Then you have two possible minimum points $(0, 0, \pm 1)$.

Comparing the functions values at these four points, you have $f(0, 0, 1) = f(0, 0, -1) = 1$ and $f(1, -1, 0) = f(-1, 1, 0) = 2$. Thus $(0, 0, 1)$ and $(0, 0, -1)$ are the points closest to the origin.

Problem 2 (5 points): Calculate

$$\iint_D y \sin(x + y) dA,$$

where $D = \{(x, y) : -\pi \leq x \leq 0, 0 \leq y \leq \pi\}$.

Solution:

$$\begin{aligned} & \int_0^\pi \int_{-\pi}^0 y \sin(x + y) dx dy \\ = & \int_0^\pi -y \cos(x + y)\Big|_{x=-\pi}^{x=0} dy \\ = & \int_0^\pi -y \cos y + y \cos(y - \pi) dy \\ = & -2 \int_0^\pi y \cos y dy \\ = & -2(y \sin y + \cos y)\Big|_0^\pi \\ = & 4 \end{aligned}$$