MATH 234 QUIZ 09 NOVEMBER 2014
NAME: TA: YUAN LIU

Problem 1 (5 points): Let V be the volume of the region in the first octant under the graph of $z = xy$ above the domain

$$D = \{(x, y) : x \geq 0, y \geq 0, x^2 + y^2 \leq 4\}.$$

Use Polar Coordinates to compute V.
Solution:

$$\begin{aligned} & \int_0^{\pi/2} \int_0^2 r\sin\theta r\cos\theta r dr d\theta \\ = & \int_0^{\pi/2} \sin\theta\cos\theta \frac{r^4}{4}\Big|_{r=0}^{r=2} d\theta \\ = & 4\int_0^{\pi/2} \sin\theta\cos\theta d\theta \\ = & \frac{\sin^2\theta}{2}\Big|_0^{\pi/2} \\ = & 2 \end{aligned}$$

Problem 2 (5 points): Find the area of the region D bounded by $y = x^2$ and $y = x + 2$.
Solutions: First, find the intersection points of $y = x^2$ and $y = x + 2$. Pluging the second equation into the first one, we have

$$x^2 = x + 2 \Longleftrightarrow (x + 1)(x - 2) = 0.$$

So we have the two intersection points: $(-1, 1)$ and $(2, 4)$. Then

$$\begin{aligned} Area &= \int_{-1}^2 \int_{x^2}^{x+2} dydx \\ &= \int_{-1}^2 x + 2 - x^2 dx \\ &= \frac{x^2}{2} + 2x - \frac{x^3}{3}\Big|_{x=-1}^{x=2} \\ &= \frac{9}{2} \end{aligned}$$