MATH 234 QUIZ 10 NOVEMBER 2014

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Problem 1 (5 points): Find the average y-coordinate on the part that lies above the x-axis of the circle with radius R and center at the origin.

Solution:

First, give a parametrization of the curve: $\vec{x}(t) = (R\cos t, R\sin t)$, $0 \leq t \leq \pi$. Thus, $\|\vec{x}'(t)\| = R$. The average y-coordinate is given by $\frac{\int_C ydx}{\int_C 1dx}$.

$$\begin{aligned}\int_C yds &= \int_0^\pi R\sin tRdt \\ &= -\cos t|_0^\pi R^2 \\ &= 2R^2\end{aligned}$$

$$\begin{aligned}\int_C 1ds &= \int_0^\pi Rdt \\ &= Rt|_0^\pi \\ &= \pi R\end{aligned}$$

Thus, the average y-coordinate is $2R^2/\pi R = 2R/\pi$.

Problem 2 (5 points): Find the moment of inertia about a diameter of a solid sphere of radius a with density function $\mu(x,y,z) = 1$.

Solutions Let the sphere at the center $(0,0,0)$ and choose z-axis to compute the moment of inertia. Then, we have

$$\begin{aligned}I_z &= \iiint_D (x^2+y^2)\mu(x,y,z)dV \\ &= \int_0^a \int_0^{2\pi} \int_0^\pi (\rho^2\sin^2\phi\cos^2\theta + \rho^2\sin^2\phi\sin^2\theta)\rho^2\sin\phi d\phi d\theta d\rho \\ &= \int_0^a \int_0^{2\pi} \int_0^\pi \rho^4\sin^3\phi d\phi d\theta d\rho \\ &= \int_0^a \rho^4 d\rho \int_0^{2\pi} d\theta \int_0^\pi \sin^3\phi d\phi \\ &= \frac{a^5}{5}\cdot 2\pi\cdot [\frac{1}{3}\cos^3\phi - \cos\phi]\Big|_0^\pi \\ &= \frac{8\pi}{15}a^5\end{aligned}$$