MATH 234 QUIZ 11 DECEMBER 2014

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Problem 1 (5 points): Compute

$$\oint_C \vec{\nabla}(x^2y^2) \cdot \vec{T}\ ds$$

where C is the counter clockwise traversed boundary of the region R defined by $x^2 + y^2 < 16$.

Solution: Since the vector field is the gradient of a function $f(x, y) = x^2y^2$ and the curve C is a closed curve, we have the line integral is zero.

Problem 2 (5 points): Apply Green's Theorem to evaluate

$$\oint_C (3y\ dx + 2x\ dy)$$

where C is the counter clockwise traversed boundary of the region $0 \le x \le \pi$, $0 \le y \le \sin x$.

Solution:

$$\begin{aligned}
& \oint_C (3y\ dx + 2x\ dy) \\
= & \iint_R (2-3)\ dA \text{ (By Green's Theorem)} \\
= & -\iint_R\ dA \\
= & -\int_0^{\pi}\int_0^{\sin x}\ dy\ dx \\
= & -\int_0^{\pi} y\Big|_0^{\sin x}\ dx \\
= & -\int_0^{\pi} \sin x\ dx \\
= & \cos x\Big|_0^{\pi} \\
= & -2
\end{aligned}$$