

Problem 1 (10 points): Let $\vec{a} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$. Compute

1. $\vec{a} + 3\vec{b}$

2. $\vec{a} \cdot \vec{b}$

3. $\vec{a} \times \vec{b}$

4. Let $\vec{c} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. What's the volume of the parallelepiped spanned by \vec{a} , \vec{b} and \vec{c} .

Solution:

1. $\vec{a} + 3\vec{b} = \begin{pmatrix} 7 \\ 3 \\ 8 \end{pmatrix}$

2. $\vec{a} \cdot \vec{b} = 1 \times 2 + 3 \times 0 + 2 \times 2 = 6$

3. $\vec{a} \times \vec{b} = \begin{pmatrix} 6 \\ 2 \\ -6 \end{pmatrix}$

4. $\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 0 & 0 \\ 2 & 2 & 0 \end{vmatrix} = 6$. So the volume is $|\vec{a} \cdot (\vec{b} \times \vec{c})| = 6$