MATH 234 QUIZ 02 SEPTEMBER 2014
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Problem 1 (3 points): Find the length of the curve

$$\vec{x}(t) = (2+t)\vec{i} - (t+1)\vec{j} + t\vec{k}, \quad 0 \le t \le 3.$$

Solution:

$$\vec{x'}(t) = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

So, the length$= \int_0^3 \sqrt{1^2 + (-1)^2 + 1^2} dt = 3\sqrt{3}$.

Problem 2 (7 points): Find the points on the curve

$$\vec{x}(t) = (5\sin t)\vec{i} + (5\cos t)\vec{j} + (12t)\vec{k}$$

at a distance 26π units along the curve from the point $(0, 5, 0)$. (There should be two such points.)

Solutions:

$$\vec{x'}(t) = \begin{pmatrix} 5\cos t \\ -5\sin t \\ 12 \end{pmatrix}$$

Then the arc length from the point $(0, 5, 0)$ to the point at $t = T$ is

$$\int_0^T \sqrt{(5\cos t)^2 + (-5\sin t)^2 + 12^2} dt = 13T.$$

We want $|13T| = 26\pi$, so $T = 2\pi$ or -2π.
So the two points satisfying the conditions are $\vec{x}(2) = (0, 5, 24\pi)$ and $\vec{x}(2) = (0, 5, -24\pi)$.