

NAME:

TA: YUAN LIU

Problem 1 (3 points): Classify the following quadratic forms:(You need to show the reasons.)

1. $x^2 + 4x + 4$

2. $10x^2 + xy$

3. $6x^2 + xy - 12y^2$

Solution: Check $4AC - B^2$

1. $4 \times 1 \times 4 - 4^2 = 0$. Semidefinite.

2. $4 \times 10 \times 0 - 1^2 < 0$. Indefinite.

3. $4 \times 6 \times (-12) - 1^2 < 0$ Indefinite.

Problem 2 (7 points): Give the function of curvature $\kappa(t)$ and normal vector $\vec{N}(t)$ for the curve $\vec{x}(t) = (2t)\vec{i} + (4t^2)\vec{j}$.

Solution: $\vec{x}'(t) = \begin{pmatrix} 2 \\ 8t \\ 0 \end{pmatrix}$ and $\vec{x}''(t) = \begin{pmatrix} 0 \\ 8 \\ 0 \end{pmatrix}$. So unit tangent $\vec{T}(t) = \frac{\vec{x}'}{\|\vec{x}'\|} = \begin{pmatrix} 1/\sqrt{1+16t^2} \\ 4t/\sqrt{1+16t^2} \\ 0 \end{pmatrix}$.

Then we have

$$\begin{aligned} \vec{\kappa}(t) &= \frac{1}{\|\vec{x}'\|^2} (-(\vec{x}'' \cdot \vec{T})\vec{T} + \vec{x}'') \\ &= \frac{1}{(2\sqrt{1+16t^2})^2} \left(-\frac{32t}{\sqrt{1+16t^2}} \begin{pmatrix} 1/\sqrt{1+16t^2} \\ 4t/\sqrt{1+16t^2} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 8 \\ 0 \end{pmatrix} \right) \\ &= \frac{1}{(1+16t^2)^2} \begin{pmatrix} -8t \\ 2 \\ 0 \end{pmatrix} \end{aligned}$$

So curvature $\kappa(t) = \|\vec{\kappa}(t)\| = \frac{2}{(1+16t^2)^{3/2}}$ and normal vector $\vec{N}(t) = \frac{\vec{\kappa}}{\|\vec{\kappa}\|} = \frac{1}{\sqrt{16t^2+1}} \begin{pmatrix} -4t \\ 1 \\ 0 \end{pmatrix}$