MATH 234 QUIZ 04 SEPTEMBER 2014

NAME: TA: YUAN LIU

Problem 1 (4 points): Find the largest domain on which the following functions of two variables can be defined:

1. $f(x, y) = \sqrt{9 - x^2} + \sqrt{y^2 - 4}$
2. $f(x, y) = \arcsin(x^2 + y^2 - 2)$
3. $f(x, y) = \sqrt{xy}$
4. $f(x, y) = \sqrt{16 - x^2 - 4y^2}$

Solution:

1. $-3 \leq x \leq 3$ and $y \leq -2$; or $-3 \leq x \leq 3$ and $y > 2$
2. $1 \leq x^2 + y^2 \leq 3$
3. $x \geq 0$ and $y \geq 0$; or $x \leq 0$ and $y \leq 0$
4. $x^2 + 4y^2 \leq 16$

Problem 2 (6 points): Let $f(x, y) = \frac{\ln(x)\sin(y^2)}{3x^3y}$. Find the tangent plane to the graph $z = f(x, y)$ at the point $(1, 1, 0)$ and a normal vector to the tangent plane. If x and y are functions of t given by $x(t) = t$, $y(t) = t^2$, find the derivative $df(1)/dt$.

Solution:

$$f_x(x, y) = \frac{\frac{1}{x}\sin(y^2)3x^3y - \ln(x)\sin(y^2)9x^2y}{(3x^3y)^2},$$

$$f_y(x, y) = \frac{\ln(x)\cos(y^2)2y3x^3y - \ln(x)\sin(y^2)3x^2}{(3x^3y)^2}.$$

So, by plugging in $x = 1, y = 1$, $f_x(1, 1) = \sin(1)/3$ and $f_y(1, 1) = 0$. Thus, the tangent plane to the graph $z = f(x, y)$ at the point $(1, 1, 0)$ is given by $z = 0 + \frac{\sin 1}{3}(x - 1) + 0(y - 1) \Leftrightarrow 3z = \sin(1)(x - 1)$, and this gives a normal vector $\begin{pmatrix} \sin(1)/3 \\ 0 \\ -1 \end{pmatrix}$.

Since $x(t) = t$ and $y(t) = t^2$, $x'(t) = 1$ and $y'(t) = 2t$. Then we have

$$\begin{aligned} \frac{df}{dt}(1) &= f_x(x(1), y(1))x'(1) + f_y(x(1), y(1))y'(1) \\ &= \frac{\sin(1)}{3}. \end{aligned}$$